物流系统分析
Logistics System Analysis

第 15 周　多到多配送问题（2）
Many-to-Many Distribution with Transshipments — Multi-Terminal Systems, One Transshipment

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- This lecture discusses systems with more than one terminal, and for the most part it will address strategic problems. We seek the location and number of terminals that should be operated, as well as the schedules and routes to be used.
- This lecture considers the case where each item is transshipped at most once, and the next lecture multiple transshipments. Symmetric strategies will be examined in some detail, with discriminating customer treatments discussed only briefly.
- We begin with an extended discussion of the operational problem – that of determining the vehicle and item routes for given terminal locations and dispatching frequency — as it is of central importance with multiple terminals.

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- A building block toward tactical and strategic analyses, the solution to the operational problem is also of intrinsic interest to public carriers. Because public carriers do not haul their own freight, they cannot determine precisely the value of the items moving through their system and the ensuing inventory costs.
- Thus, for these carriers the tactical problem is somewhat academic. In practice the service level (e.g.daily deliveries) is chosen based on marketing considerations, and is widely advertised. The market then determines which types of commodities move through the system.

- The routing schemes about to be introduced extend those in Hall (1984), Hall and Daganzo (1984), Daganzo (1987c), and Campbell (1990b). For clarity, they are described for a one-dimensional region first, with 2-dimensional generalizations introduced later. For the one-dimensional case we describe non-hierarchical solutions –where the same flow is routed through all the terminals –first, and more efficient hierarchical methods second.

Non-hierarchical routing on the line

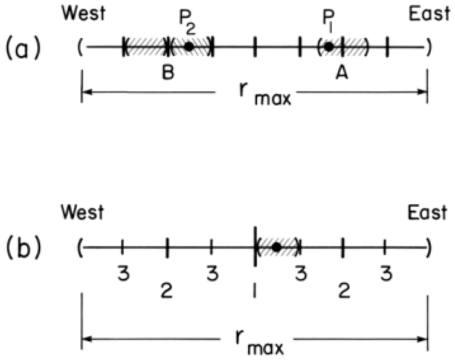


Figure a displays a region $\boldsymbol{R}$ and $N_T = 7$ evenly spaced terminals. We assume that there are many origins and destinations in the region $(N_o, N_d \ll N_T.)$

- The non-redundancy principle introduced in 1-to-N problems w/ transshipment for one-to-many networks also applies here; with only one transshipment allowed, the flow between each O-D pair should move through only one terminal. As a result each terminal has a separate set of origin-destination pairs to serve.
- Given this set, each terminal should be operated as studied in the previous section. We will assume (reasonably so) that all the origins and destinations are served with the same headway H.
- As a result the number of stops made by vehicles on their peddling and collecting routes must be adjusted by location in response to the spatially changing demand and supply rates.

- The terminal could then be operated on a clock, with all the vehicles arriving and leaving the terminal at once, for minimal delays to the items. We will also assume that H is the same for all terminals. This is reasonable because a unique H simplifies the operating plan and the job of advertising the service schedules.
- The best operating plan will minimize the total vehicle-miles and the number of vehicle stops. We assume that pipeline inventory can be neglected, and that vehicles leave and arrive at the terminals full

- Assuming that each origin generates less than a truckload of goods per headway, the goods it ships through a terminal can be collected with a single stop by a single collecting vehicle. Hence, the number of collection stops made during H at one origin m^o equals the number of terminals to which that origin is shipping. Similarly, the number of delivery stops per destination m^d is the number of terminals from which deliveries are received.
- The number of stops made in H is a direct function of the allocation of O-D pairs to terminals. In a subregion (interval) of unit size (length) the number of stops is: $\delta^o m^o + \delta^d m^d$

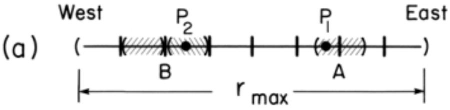
- As in describing the non-detailed vehicle routing problem, we define collection (distribution) line-haul distance of a terminal as the average distance to (from) the terminal from (to) every origin (destination) using it, multiplied by the number of collection (distribution) tours started at the terminal.
- In other words, *the total line-haul distance in* $\boldsymbol{R}$ *equals the number of item-miles traveled, divided by* v_{max}. *Therefore, it is uniquely defined by the allocation of O-D pairs to terminals*.

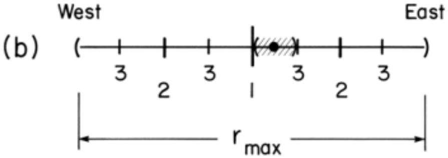
- Note that if each vehicle were to make only one collection stop, then the line-haul distance would equal the total distance traveled. Because vehicles make multiple stops, the total distance traveled is greater than the line-haul distance.
- In agreement with the NVRP, we call the distance added by the stops "local distance".

- We now show that the local collection distance traveled per headway in a given region is proportional to the number of stops made in the region, except for a constant that can be ignored. First note that the local distance for a tour with n_s stops is: $(n_s - 1)/(2\delta^o)$.
- This is true because for every two stops added to a tour, its length only increases by one interstop distance, $(\delta^o)^{-1*}$.
- Clearly, according to the formula, each collection stop made in a region contributes $(2\delta^o)^{-1}$ distance units to the total local distance, and each vehicle tour subtracts the same amount from this total. Because the total number of collection tours is fixed (remember that vehicles travel full) the total distance deducted in $\boldsymbol{R}$ is a constant, which we ignore here. The same occurs for distribution, where each stop adds $(2\delta^d)^{-1}$ distance units.
- This establishes that the local travel costs, for both collection and distribution, only depend on the number of stops made in the region; the less the better.

*We may imagine that one stop is tacked on to the end of the tour and the other one to the beginning, so as to keep the tour's center of gravity fixed; then only the stop at the far end lengthens the tour.

- Since the number of stops is a direct function of the O-D allocation to terminals, an allocation uniquely defines the local travel costs; as well as all the stopping and line-haul travel costs.
- In the following we examine various allocation strategies and their effect on stops, local distance and line-haul distance. Since local distance is proportional to the number of stops, to assess the efficacy of an operating strategy, it suffices to keep track of the line-haul miles and the number of vehicle stops





A possible strategy is depicted by the arrangement in Figure(a). The shaded area around terminal "A" represents its collection influence area. We assume that all the items from the shaded area are shipped through "A", regardless of destination, and that origins outside the shaded area ship through other terminals.

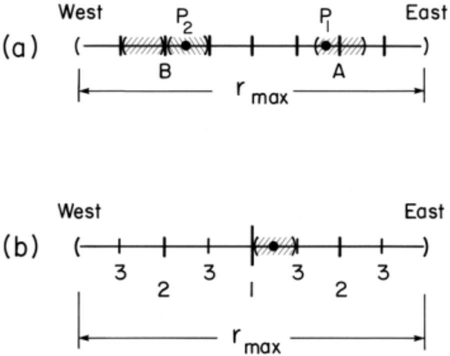
- Consequently, from that terminal items are delivered to all destinations. If all the terminals in the figure were to operate in this manner the influence areas would partition the service region and, we would have: $m^o = 1$ and $m^d = N_T$; thus, the number of stops per headway, per unit length would be: $\delta^o + \delta^d N_T$.
- If there are more destinations than origins then it would be better to define *distribution* influence areas and the number of stops would be smaller: $\delta^o N_T + \delta^d$; we will assume without loss of generality that this is not the case.
- The strategy we have just described is termed 1-terminal routing because each (small) area either ships or receives from only one terminal. A drawback of the strategy is that items sometimes travel more line-haul miles than the minimum possible, as happens for an item traveling from P1 to P2 in Figure a.

- An alternative routing scheme that eliminates this backtracking is illustrated with terminal "B". This terminal has two influence areas, displayed by the cross-hatched segments to its right and left, but it only draws part of the supply from these areas. The influence area located to the left of B ships through B only items destined for points east of B (as well as for 1/2 of the points in the influence area that are closer to B than to any other terminal.) The influence area located to the right, similarly, sends items to all points west, and to the points within itself that are closest to B.
- This 2-terminal routing scheme eliminates back-tracking for most origin-destinatio pairs, except for O-D pairs lying entirely within two neighboring terminals. The ensuing savings in line-haul distance are achieved at the expense of one extra stop per origin. Since $m^o = 2$, the number of stops per headway and per unit length is now: $2\delta^o + \delta^d N_T$.

- In going from 1- to 2-terminal routing we save approximately $r_{\max}/4N_T$ line-haul vehicle-miles per inbound vehicle tour (since in the first case about $1/2$ of the inbound miles are backtracking miles and in the second case nearly none), but we add δ^o stops per unit length per headway. If the level of demand is such that we require N_v vehicle tours to collect all the items in $\boldsymbol{R}$ during a headway ($N_v = \lambda r_{\max}^2 H/v_{\max}$) then the saved line-haul vehicle-miles per headway in $\boldsymbol{R}$ are: $r_{\max}N_v/4N_T$. Usually, $N_v \gg N_T$, and the total line-haul distance saved should be several times larger than $r_{\max}$.
- The extra local collection distance, on the other hand, is negligible by comparison since it equals: $(\delta^o)r_{\max}(2\delta^o)^{-1} = r_{\max}/2$. Thus, only if the intrinsic cost of a stop, c_s, is large enough to nullify the line-haul savings, would the 1-terminal strategy be preferable. This is a moot issue, however, because the number of stops can be reduced below the 1-terminal levels, *without additional backtracking*, by hierarchical schemes.

Hierarchical routing on the line

- So far, as in Hall (1984) and Hall and Daganzo (1984), terminals have not been differentiated in any manner; if the origin and destination flows don't change much with location, the flow passing through each terminal is nearly the same. These strategies, however, result in many more delivery than collection stops, or the opposite.
- We illustrate now how one can greatly decrease the number of delivery stops with a small increase in the number of collection stops.



- Figure (b) shows the same region and terminals of (a), but now the terminals have been labeled by numbers. The terminal near the center is labelled “1”; it partitions R into two equal halves. The terminals located near the middle of each half are labelled ""“2”, and the ones located near the middle of each fourth are labelled “3”. These labels represent levels within a hierarchy of terminals, with “1” being the highest level.
- A system with a full set of terminals and $l = 1, \ldots, L$ levels will have $2^L - 1$ terminals and $2^{(l-1)}$ terminals at each level. (In earlier texts L denoted the number of time periods, but that variable is not used here.)

- A routing strategy that avoids backtracking could be defined as follows

Serve each O-D pair by the highest level terminal between the origin and the destination; if there is no terminal in between, use the neighboring one which can be reached from both customers with the least combined distance.

The definition uniquely identifies a terminal for each OD pair; because there can never be a tie for the highest level between terminals. This is true because, with our labelling strategy, two terminals of the same level are always separated by one or more higher level terminals.

- Notice that each origin sends items exactly through L terminals and receives items through L terminals. For example, origins in the crosshatched section of Figure (b) would ship through the level-1 terminal for all points west of the section, through the level-3 terminal at the right end of the section for points in the neighboring section to the right, and through the level-2 terminal on the right half of the region for the remaining points farther east. The destinations in the shaded region would also receive items from the whole region through the same three terminals. (It is recommended at this point to identify mentally the 3 terminals that would be used for each of the 8 segments in the figure).

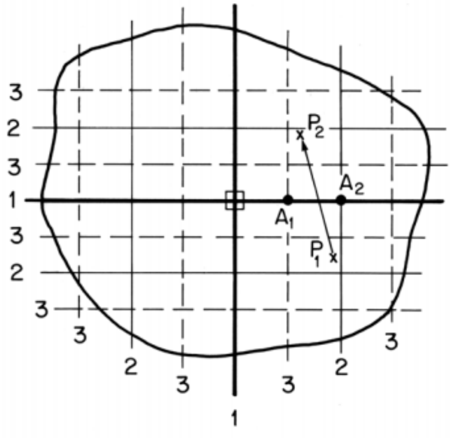
Not given in detail here, a formal proof of our statement for arbitrary L can be constructed along the following lines.

- For any origin segment, one would start by identifying the set of destinations served through the level-1 terminal. Recognizing that lowerlevel terminals within this set are not used, one would then show that only one of the level-2 terminals is used to serve the remaining points. The argument would then be repeated for lower levels.
- Thus, without increasing line-haul (backtracking) miles, the number of stops can be reduced to $L(\delta^o + \delta^d)$ from $2\delta^o + \delta^d N_T$. If $\delta^o \approx \delta^d$, the reduction in the number of stops can be quite substantial: from $m^o + m^d \approx N_T$ to $m^o + m^d \approx 2L = 2\log_2(N_T + 1)$. Thus, we see that with a hierarchical routing strategy, the number of stops only increases logarithmically with the number of terminals.

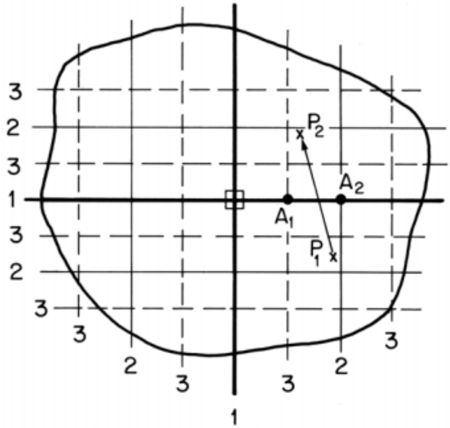
- Note that the flows through the various terminals are radically different even if the origins and destinations are uniformly distributed. Not counting O-D pairs within a segment, the level-1 terminal handles $1/2$ of all the origin destination pairs. The level-2 terminals handle $1/2$ of the rest; i.e. $1/4$ of the total.
- Since there are two level-2 terminals, each handles $1/8$ of the total. Assuming that there are more than 3 levels, the level-3 terminal would handle $1/2$ of the rest: $(1/2)^l$ (for $l = 3$), and since there are $2^{(l-1)}$ terminals of this type, each would handle $2/(4^l)$ of the traffic, etc...

- Hierarchical terminal systems are used by many common carriers.
- Federal Express, an overnight package delivery carrier (H=1 day), started their operation with one hub in Memphis (Tennessee) and later opened another hub in Oakland(California). The Oakland hub is a secondary hub that only serves O-D pairs in the Western United States, and is in our terminology a level-2 terminal. Federal Express operates nowadays with L=2.
- Similar hierarchies can be found upon inspection of airline networks, although in that case the highest level terminals cannot carry as much traffic as it would be ideal because of airport capacity limitations.

Two-dimensional extension



- We are now ready to see how the hierarchical strategy can be extended to two dimensions. In this case it helps to think in terms of two sets of parallel lines in two perpendicular directions, defining a square grid as shown in figure. Each set of lines is numbered with the bisecting strategy used in figure (b) in the one-dimension case. The dark (level-1) lines should cross near the center of the region, ***R***, and terminals are assumed to be located at or near the intersection of any two lines (level-3 lines are represented by dashed lines in our figure). Thus, with L=3, there should be a maximum of $(2^l-1)^2$ terminals, since there are (2^l-1) lines in each family. The actual number of terminals may be smaller if some of the lines intersect outside ***R***


3
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P_2
A_2
A_1
P_1

- The terminal selection process for a given O-D pair is simple. Choose the highest numbered line from each set that is crossed by the segment joining the origin and the destination, and use the terminal located at the point of intersection.
- As in one dimension, this defines unambiguously the terminal to be used, unless the trip does not cross a line in one of the directions. If this happens, one is assumed to choose the least circuitous terminal on the highest level line crossed in the other direction; thus, in traveling from P_1 to P_2 an item would be shipped either through A_1 or A_2.
- If the path crosses no lines, then the origin-destination pair lies entirely within a cell of the grid and one would choose among the four terminals on the corners

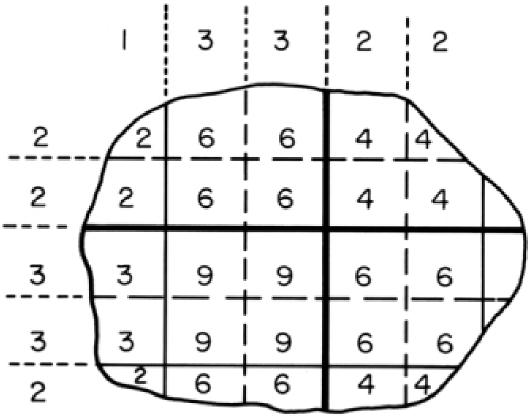
- If travel were only possible in the directions of the grid (distances follow an L_1 metric) then only the trips in which one (or both) of the families are not crossed would entail some back-tracking. If the origins and destinations are independently distributed of each other, and $N_T \gg 1$, then the probability that a trip requires some backtracking in one direction will be on the order of $(1/N_T)^{1/2}$, the reciprocal of the number of lines in one direction; and the average distance added to the trip will be about $1/3$ of the separation between terminals, $(|\boldsymbol{R}|/N_T)^{1/2}$.
- This extra distance result holds because: (i) the sum of the distances to the best line (of the two possible) is $2/3$ of the distance separation between lines, and (ii) because as is well known (e.g., Larson and Odoni, 1981), the average separation between points is $1/3$ of the lattice spacing. Thus, the expected added distance is one third of the lattice spacing, as claimed; and if one considers both directions the incremental distance should be twice as large.

- Since the probability of backtracking in either direction is $1/N_T$, the expected added distance across all O-D pairs should then be:

$$2|\boldsymbol{R}|^{1/2}/(3N_T)$$

- This expression assumes that travel takes place along a grid. If this is not the case the distance added by the terminal stopover will be a different expression, but should behave qualitatively similarly.
- The expression used in Daganzo (1987c), $[2|\boldsymbol{R}|^{1/2}/(3N_T)][1-(4N_T)^{-1/2}]$, is qualitatively similar to this expression. Developed by Hall and Daganzo (1984) for a 4-terminal routing strategy (which is inefficient in terms of number of stops but yields the same backtracking distance), this expression is exact when $\boldsymbol{R}$ is a square; it accounts for the peculiar edge and corner zones, which is only important if N_T is small.

Hierarchical routing without a full complement of terminals



- If $\boldsymbol{R}$ is not close to a square, or N_T is not close to $(2^L - 1)^2$ for some integer L, then some levels may have less than a full complement of terminals. As illustrated in the previous page; with $L = 3$, it has $N_T = 18$ terminals, when N_T should have been 49. It should be clear from the derivation, however, that expression for added distance* should be fairly accurate even without a full complement of terminals.

*$2|\boldsymbol{R}|^{1/2}/(3N_T)$

- Let us now turn our attention to the number of stops per origin and destination, m^o and m^d. With a full complement of terminals, each origin would ship through L^2 terminals, and nearby destinations would receive through the same terminals; thus, $m^o \approx m^d \approx L^2$. With less than a full complement of terminals, the number of stops would be smaller.
- The figure depicts the number of stops for collection (or delivery) that are made per origin (or destination) in each cell. Note that, even though $L = 3$, only 4 cells require 9 stops. The average across cells is significantly smaller, approximately 6.7 stops.

- reasonable approximation for the number of stops is given by L^2, using for L the real solution of $N_T = (2^L - 1)^2 : L = \log_2(1 + N_T^{1/2})$. That is:

$$m^o = m^d \approx [\log_2(1 + N_T^{1/2})]^2$$

- For $N_T = 18$, as in the figure, this yields a better approximation than using $L = 3$; i.e., 5.7 , instead of 9 stops. This expression is exact if the amount in brackets is an integer, and other examples (e.g. with $N = 4$ and $N = 12$) reveal that it tends to under-predict the actual number of stops by about 10%. A simpler expression which is very accurate for $N_T < 10^2$ is:

$$m^o = m^d \approx 2.6 N_T^{1/2}$$

  and since these two equations tend to under predict the actual average by about 10% we will use instead: $m^o = m^d \approx 3 N_T^{1/2}$

- We notice that the circuity distance decreases with N_T, but the cost caused by local stops increases with N_T.
  - circuity distance$=2|\boldsymbol{R}|^{1/2}/(3N_T)$, costs caused by local stops$= m^o = m^d \approx 3N_T^{1/2}$
- Thus, we shall look for the number of terminals that minimizes cost. Before we address this strategic problem, one last point needs to be discussed.

Detailed solution

- It has been assumed so far that terminals were more or less located on a square lattice within the service region. We then showed how it was possible to develop a labeling system that minimized the number of terminals serving each point in $\boldsymbol{R}$ while keeping backtracking at a minimum.
- If the terminal locations are given and they do not remotely resemble a lattice, one can achieve the same goal with a detailed trip assignment scheme. Essentially, each O-D pair (i, j) must be assigned to one terminal, $k = 1, \ldots, N_T$, minimizing the line-haul distance cost and, the local motion cost–including local distance and stops.

- The solution can be specified in terms of zero-one decision variables x_k^{ij}, taking the value 1 if terminal k is used between i and j, and 0 otherwise. The line-haul cost equals, as before, the item-miles traveled multiplied by $c_d/v_{\max}$, and each local stop adds $\alpha_2 \approx (c_s + c_d k \delta^{1/2})$ to the local motion cost. (Note that the distance arguments given for one-dimensional problems were equivalent to using $k = 0.5$ in the expression for α_2).
- Letting r_k^{ij} denote the known distance of a trip from i to j, passing through terminal k, and D^{ij} the number of items that must be carried from i to j during one headway, we can write:

$$\text{total \# stops} = \sum_{ik} \min\{\sum_j x_k^{ij}; 1\} + \sum_{jk} \min\{\sum_i x_k^{ij}; 1\}$$

and

$$\text{total \# item-miles} = \sum_{ijk} D^{ij} r_k^{ij} x_k^{ij}.$$

- Assuming for simplicity that α_2 is the same for pickups and deliveries ($\delta^o \approx \delta^d$) , then we would like to minimize:

$$\left[\sum_{ik} \min\left(\sum_j x_k^{ij}; 1\right) + \sum_{jk} \min\left(\sum_i x_k^{ij}; 1\right)\right] + \left(\frac{c_d}{v_{\max}}\right) \sum_{ijk} D^{ij} r_k^{ij} x_k^{ij}.$$

where the x_k^{ij} are zero-one variables with:

$$\sum_k x_k^{ij} = 1.$$

- It is not difficult to include terminal flow restrictions (e.g. requiring that $\sum_{ij} D_{ij} x_k^{ij}$ remains below some limit for terminal k) in the formulation, but this may complicate the solution procedure

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- To solve the strategic and tactical problems we use the optimal solution of the idealized operational problem. This is reasonable, since at this level one should not plan to use a poor set of terminal locations.
- The collection cost per item transported due to the number of stops is based on $m^o + m^d = 3N_T^{1/2}$. Recognizing that the number of stops at each origin is $m^o = 1.5N_T^{1/2}$, we can write:

$$\text{inbound stop/item} = \alpha_2 \left(\frac{\delta^0}{\lambda H |\boldsymbol{R}|} \right) 1.5N_T^{1/2})$$

  where the quantity in brackets represents the reciprocal of the number of items collected at the average origin in one headway. A similar expression holds for the outbound stop cost.
- If we assume that $\delta^o = \delta^d = \delta$ (the reader can generalize this assumption easily) we obtain:

$$\text{stop cost/item} = \frac{3\alpha_2\delta}{\lambda H |\boldsymbol{R}|} N_T^{1/2}$$

- We will also assume in our exposition that λ and δ do not change over the region. If they do it is better to work with a (less intuitive) total regional cost per day; the resulting expressions, presented in the extension discussions on the multi-terminal systems with multiple transshipments, are close to the ones with constant conditions when averaged across items. Qualitatively similar conclusions are reached. A line-haul circuity cost per item can be obtained from the extra distance traveled by each item $2|\boldsymbol{R}|^{1/2}/(3N_T)$. Since items travel in full vehicles, the prorated circuity cost per item is:

$$\text{circuity cost/item} = \frac{c_d}{v_{\max}} \times \frac{2|\boldsymbol{R}|^{1/2}}{3N_T}.$$

- This cost is paid in addition to the basic line-haul cost, which is proportional to the average distance between origins and destinations; this basic cost is of order $[c_d/v_{\max}]|\boldsymbol{R}|^{1/2}$.

- We should also include a fixed cost of operating a terminal*, with $|\boldsymbol{R}|/N_T$ instead of I, which should increase linearly with N_T:

$$\text{terminal cost/item} = \alpha_5 + \alpha_6 \frac{N_T}{|\boldsymbol{R}|}$$

- Finally, we must also include the stationary holding cost at the origins and destinations:

$$\text{holding cost/item} = c_h H$$

- The sum of stop cost, circuity cost, terminal cost and holding cost is our logistic cost. With it we can answer a variety of questions. A strategic level question could be: how many terminals should be operated, given H? This might be appropriate for a carrier that is planning entry in a market niche with a well defined H.

*terminal cost/item $\approx \alpha_5 + \alpha_6/I$

- Alternatively we may be interested in determining the best H for a given N_T, or in selecting both together. Everything is possible, and easy to do, since the objective function is defined in terms of only one or two decision variables, and is unimodal (it is a "positive" polynomial of the form used in geometric programming).
- For a given N_T, the best H balances local stop costs and holding cost; circuity and terminal costs are fixed. We find:

$$H^* \approx \left[\frac{3\alpha_2 \delta}{\lambda c_h |\boldsymbol{R}|} N_T^{1/2} \right]^{1/2}$$

and the total cost per item, not including the fixed basic line-haul cost, is:

$$\begin{aligned} \text{cost/item} =& \alpha_5 + \alpha_6 \frac{N_T}{|\boldsymbol{R}|} + \frac{c_d}{v_{\max}} \times \frac{2|\boldsymbol{R}|^{1/2}}{3N_T} + 2\left(\frac{3\alpha_2 \delta c_h}{\lambda |\boldsymbol{R}|} N_T^{1/2} \right)^{1/2} \\ =& \alpha_5 + \alpha_6 \frac{N_T}{|\boldsymbol{R}|} + \frac{c_d |\boldsymbol{R}|^{1/2}}{v_{\max}} \times \left[\frac{2}{3N_T} + 2\left(\frac{3\alpha_2 \delta c_h v_{\max}^2}{\lambda |\boldsymbol{R}| c_d^2} \right)^{1/2} N_T^{1/4} \right]. \end{aligned}$$

- As an example, we find the optimal number of terminals for a case where fixed terminal costs can be neglected and where the cost of a stop c_s is small compared to the distance component $kc_d\delta^{-1/2}$; thus $\alpha_6 = 0$ and $\alpha_2 \approx c_d k\delta^{-1/2}$. The cost per item (using $(3k^2)^{1/2} \approx 1$ and disregarding the constant α_5) is:

$$\text{cost/item} \approx 2\left(\frac{c_d|\boldsymbol{R}|^{1/2}}{v_{\max}}\right)\left(\frac{1}{3N_T} + \frac{K}{N_0}N_T^{1/4}\right)$$

where N_o is the number of origins (and destinations), and K is the dimensionless constant introduced at the outset of lecture.
    - the optimal number of stops $n_s = v_{\max}\left(\frac{\alpha_4}{\alpha_2}\right)^{1/2} = \left[\frac{c_h\delta^o\delta^d v_{\max}^2}{\lambda[c_s+c_d k(\delta^d)^{-1/2}]}\right]^{1/2} = K(\boldsymbol{x}^o, \boldsymbol{x}^d)$

  The factor in brackets, comparable with the basic line-haul cost, represents the cost of crossing the service region (if it was "round" in shape) prorated to the items in a full vehicle.

- The minimum of cost per item is obtained for

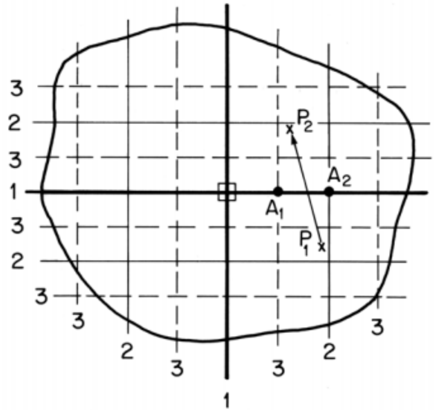
$$N_T^* \approx \left(\frac{4N_o}{3K}\right)^{4/5},$$

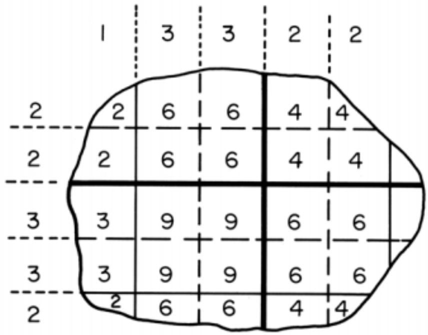
and the result is

$$\text{cost/item} = \left(\frac{c_d|\boldsymbol{R}|^{1/2}}{v_{\max}}\right)\left[2.6\left(\frac{K}{N_o}\right)^{4/5}\right]$$

- The cost without transshipments*, when expressed as a function of the same variables, adopts the same form but the term in braces is of order $K/(N_o^{1/2})$. Clearly, if N_o is large, terminals reduce cost dramatically.

$^*z_o = c_s' + \frac{\alpha_1+2\alpha_2 K}{v_{\max}}$





A comparison of the number of stops of each vehicle route is interesting. The numbers of local stops w/o transshipments is $n_s \approx K$. With transshipments, the number of stops is larger for the vehicles serving the lowest level terminals, located at the intersection of dashed lines in the figures. Thus, we focus on these.

Interpretation

- Vehicles based at one such terminal serve all the destinations in the 4 cells next to it and no destinations beyond; a total of $4\delta^d|\boldsymbol{R}|/N_T$ customers. A similar expression holds for origins. The item flow passing through the terminal is the average flow for one O-D pair, $\lambda/(\delta^o\delta^d)$, multiplied by the number of pairs served through it. The terminal can only serve O-D pairs entirely within a square 4-cell sub-region centered at the terminal. There are $16\delta^o\delta^d[|\boldsymbol{R}|/N_T]^2$ such O-D pairs. Some of these, however, are better served by terminals on the edge of the square subregion.
- Thus, the actual number served through the terminal should be somewhat smaller. We may verify that only 9/16 of the O-D pairs in the sub-region are served through the terminal if origins and destinations are uniformly distributed.

- Therefore, we can write:

$$\# \text{ OD pairs served} = 9\delta^o\delta^d\left(\frac{|\boldsymbol{R}|}{N_T}\right)^2,$$

so that the flow through a lowest level terminal is about: $9\lambda[|\boldsymbol{R}|/N_T]^2$ items per unit time, or $(9\lambda/v_{\max})H(|\boldsymbol{R}|/N_T)^2$ delivery vehicle loads (trips) per dispatch.
- Since these trips must collectively stop at $4\delta^d|\boldsymbol{R}|/N_T$ customers, the average number of delivery stops per trip is:

$$n_s^d \approx \frac{4\delta^d v_{\max} N_T}{9\lambda|\boldsymbol{R}|H}.$$

The collection stops are given by a similar formula.

- The expressions for the optimal frequency H, # terminals N_T and # delivery stops per trip

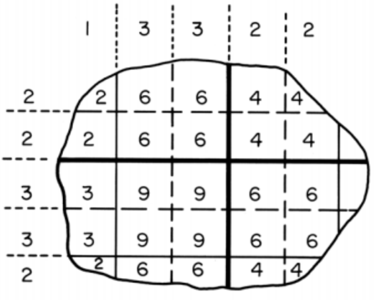
$$H^* \approx \left[\frac{3\alpha_2\delta}{\lambda c_h|\boldsymbol{R}|}N_T^{1/2}\right]^{1/2}$$
$$N_T^* \approx \left(\frac{4N_o}{3K}\right)^{4/5}$$
$$n_s^d \approx \frac{4\delta^d v_{\max} N_T}{9\lambda|\boldsymbol{R}|H}.$$

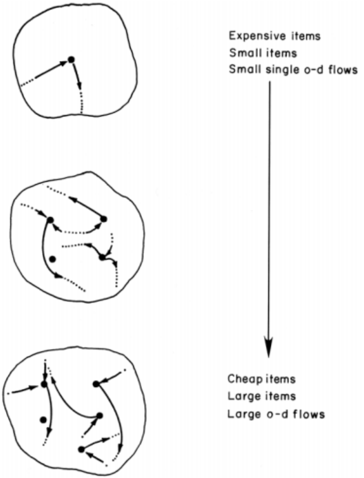
yield the following expression:

$$\frac{n_s^*}{\sqrt{N_0}} \approx 0.3\left(\frac{K}{N_o}\right)^{2/5}.$$

With many customers ($N_o > 100K$), this value is smaller than the average # of delivery stops w/ the non-hierarchical strategy in Daganzo (1987c): $\frac{n_s^*}{\sqrt{N_0}} \approx (\frac{K}{N_o})^{2/3}$

- For example, if $N_o = 10^4$ and $K = 10^2$, then one should operate about 50 terminals, using something like the 3-level pattern depicted in the figure, and vehicles would make a maximum of 5 stops. The number of stops without terminals would have been much greater, $n_s = 10^2$.
  - It follows $n_s = K$, and $z_0 = c_s' + (\alpha_1 + 2\alpha_2 K)/v_{\max}$
- With a nonhierarchical strategy the average number of delivery stops is also close to 5 but we can only use 25 terminals. Because of the increased circuity, the cost is about 20% higher.





Figure

Qualitatively, though, the results in that reference and the improved ones presented here tell the same story. As the items become more valuable, and the origins more diffuse and small the number of terminals should be reduced. Cheap bulky items can be routed through more terminals, which is logical since the circuity costs will dominate. The only difference between the hierarchical and nonhierarchical results is that the optimal system can make use of more terminals since the number of stops does not increase as rapidly with N_T.

- Note also that if No decreases but other parameters change so as to keep K constant, e.g., the individual customers become larger, the advantage of $\text{cost/item} = \left(\frac{c_d|\boldsymbol{R}|^{1/2}}{v_{\max}}\right)\left[2.6\left(\frac{K}{N_o}\right)^{4/5}\right]$ over $z_0 = c_s' + \frac{\alpha_1+2\alpha_2 K}{v_{\max}}$ also decreases. If one factors in the fixed and variable terminal costs we find that the optimal N_T is smaller; not surprisingly, shipping without a transshipment eventually becomes desirable for sufficiently large customers.

目录 |Outline

- It was assumed until now that vehicle routes could be as long as necessary and have as many stops as needed. If this is not the case, but we are still dealing with cheap items carried in full vehicles, one can modify the optimization of the strategic and tactical problem to yield the desired result.
- One would still try to run the system on a clock, with a common headway, but perhaps would stop introducing new terminals as soon as the lowest level terminals resulted in routes with too many stops.

- The desired system configuration would be given by the minimum of the sum:

$$\text{stop cost/item} = \frac{3\alpha_2\delta}{\lambda H|\boldsymbol{R}|} N_T^{1/2}$$

$$\text{circuity cost/item} = \frac{c_d}{v_{\max}} \times \frac{2|\boldsymbol{R}|^{1/2}}{3N_T}$$

$$\text{terminal cost/item} = \alpha_5 + \alpha_6 \frac{N_T}{|\boldsymbol{R}|}$$

$$\text{holding cost/item} = c_h H$$

where N_T and H would have to satisfy $n_s^d, n_s^o \leqslant n_{\max}$, with the $n_s^d \approx \frac{4\delta^d v_{\max} N_T}{9\lambda|\boldsymbol{R}|H}$. This constraint, like the inclusion of terminal costs $\alpha_5 + \frac{\alpha_6 N_T}{|\boldsymbol{R}|}$ in the objective function, will tend to produce a smaller NT than suggested by $N_T^* \approx (\frac{4N_o}{3K})^{4/5}$.

- The system of equations were developed for cheap items, and identical vehicles, but similar expressions can be developed in other cases, including situations where H can vary across terminals of different levels. Although it is impossible to cover all aspects of the problem in this monograph, it should be clear that in many cases the steps to be followed should be quite similar.
- The Problem 6.6 addresses an idealized situation peculiar to airlines (the exercise extends the work of Jeng, 1987, who studied an idealized model of a single hub airline.)

- If some of the origins and destinations are much larger than others it may be worthwhile to consider discriminating strategies whereby the origin-destination pairs with the largest flow would be served non-stop and the rest though the system of terminals, as in Sec. 6.3.2.

- If pairs are chosen for inclusion in either one of the categories based on the amount of flow alone, with no regard for location, then it is possible to find the best O-D pair allocation (and the resulting system design) by conditioning on the number of pairs that are handled without a transshipment.
- For any number, the costs on the two systems are independent of actions taken to control the other system and as a result the two can be optimized separately, as we have learned previously. A near-best allocation can be formed by repeating the process for various (carefully selected) numbers of origin-destination pairs in the non-terminal system, and comparing total costs.

- As with one terminal, ideally one might want to use the geographical locations of origins and destinations and relevant flow information in deciding where to allocate an O-D pair, but the problem is more complex than with only terminal. Fortunately, with several terminals the importance of location is diminished because the maximum distance added by a terminal stop-over is smaller.
- In summary, this section illustrates how the number of vehicle stops and the total logistic cost can be reduced by transshipping items once at breakbulk terminals. We have seen that a hierarchy of terminals enhances the transshipment benefits.

Any questions?

Readings

- Daganzo. Logistics System Analysis. Ch.6. Page 232-248.