物流系统分析
Logistics System Analysis

多到多配送问题—单中转枢纽系统
Many-to-Many Distribution — One Terminal System

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- Our previous lectures have been devoted to logistics problems involving the movement of freight and people from one origin to any number of destinations — or else to the reverse problem of collecting freight and people from any number of origins for a single destination.
- We now turn our attention to problems involving any number of origins and destinations. In practice, many-to-many problems arise in connection with public carriers such as: airlines, the postal service, less-than-truckload carriers (零担货运), railroads, etc. Unlike for private carrier operations, where most of the logistic costs are borne by the firm, some costs are now borne by the carrier and some by the shipper. 承运人和托运人都可能承担费用。
- This dichotomy （两分法）, introduced in the one-to-many problem with variable demands, will also be recognized here.

Description of the problem

- The logistic problem will be specified as before, in terms of a geographical distribution of origins and destinations with certain supply and demand rates.
- It will be assumed that **each destination demands a specific number of items from each one of the origins** and that these **cannot be substituted** for one another. That is, we are dealing here with what normally is referred to in the network optimization literature as a **multi-commodity problem**. 多商品问题

Single commodity problems

- Single commodity problems arise if destinations specify a combined demand regardless of point of origin. Examples of these are water and electricity supply problems.
- These problems allow the analyst to specify which origins ship to which destinations, which greatly reduces the need for travel and transportation

Single commodity problems (cont.)

- Single commodity problems are not addressed here because they can be reduced to special cases of the problems studied in 1-to-N problems (with/without TS).
- If there are more destinations $\{j\}$ than origins $\{i\}$, one can introduce a single "super-origin" O_o with production rate equal to the total demand rate at $\{j\}$, and then worry instead about finding the best scheme for serving $\{j\}$ from O_o through a set of "terminals," $\{i\}$, assuming that items can be moved freely from O_o to $\{i\}$.
- To ensure that each real origin ships the prescribed amounts, the capacities of the fictitious terminals should be set equal to the origin's maximum production rates.
- If there are more origins than destinations, one would seek the best way of carrying items from the origins to a fictitious super-destination D_o.

Single commodity problems (cont.)

- This one-to-many interpretation of the single commodity problem indicates that **each real origin should serve the destinations in an influence area surrounding it**, possibly with a transshipment. These influence areas partition $\boldsymbol{R}$, and the operations in the influence areas, conditioned on certain variables, should be independent of one another.
- Of course, the shed boundaries separating influence areas may shift with time if the demand and production rates vary with time. The system can be designed as described in the "Refinements and Extensions" and "Multiple Transshipments" sections in the 1-to-N problems with transshipments.

Multi-commodity problems

- Unfortunately, multi-commodity problems cannot be reduced in the same manner to a problem with a single origin or destination; they are inherently different and more difficult.
- We will focus on the aspects of the problem that are better understood, for the most part involving **stationary data and solutions**, and emphasizing problems for which **pipeline inventory is a negligible quantity**.
- Reasonable for most freight transportation problems, this emphasis may not be appropriate for many-stop passenger transportation systems, such as public transit. A thorough treatment of passenger transportation issues is beyond the scope*.

*Carlos F. Daganzo, Yanfeng Ouyang. Public Transportation Systems: Principles of System Design, Operations Planning and Real-Time Control. World Scientific. 2019.

- We may move on to a brief discussion of a new role played by terminals in many-to-many logistics systems;
- We then examines distribution without terminals and no transshipments. Followed by examining the organization of systems with only one transshipment per item

The break-bulk* role of terminals

- We have studied how transshipments allowed items from a single origin to travel long distances in large batches to a terminal, and then in small batches from the terminal to the customers. This allowed route length restrictions and delivery vehicle size limitations to be met, while preserving transportation economies of scale.
- The same economies occur in reverse: small vehicles can carry items from scattered origins to a terminal, where the small loads can be consolidated into larger ones for transportation to a destination. We can think of terminals in many-to-one and one-to-many systems as **consolidation points** that allow line-haul and local operations to be decoupled.
- Consolidation is in fact their only role, for if there are no incentives to keep local routes short (e.g. pipeline inventory considerations, delivery vehicle size limitations, or other restrictions) and a filled vehicle carries a well defined number of items, then transshipments can be shown not to reduce costs (Daganzo, 1988).

*散货运输，与 bulk(大宗货物) 相对应。

Verification of the role

- It can be verified analytically from $\underbrace{\alpha_5 + \alpha_6/I}_{\text{terminal}} + \underbrace{z^i(\lambda, r, I)}_{\text{inbound}} + \underbrace{Z^0(\lambda, \delta, I)}_{\text{outbound}}$ by:
  1. substituting either the total combined cost expression* or $z = \frac{\alpha_1}{v_{\max}} + \frac{\alpha_2}{v_{\max}} + \alpha_4 v_{\max}$ with $\bar{D}'$ replaced by λ/δ for the function $z_0(\lambda, r, \delta)$ used to calculate the outbound cost†;
  2. using the total combined cost expression with I^{-1} instead of δ for $z^i(\lambda, r, I)$.
- Simple algebraic manipulations then reveal that the minimum $z_1^*(\lambda, r, \delta)$ is greater than $z_0(\lambda, r, \delta)$ 大宗货运时，不使用中转枢纽成本更低。

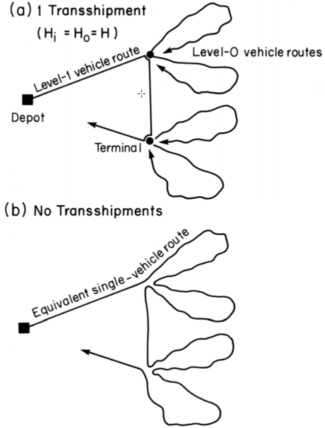
*$[c_s + 2c_d E(r)]/v_{\max} + c_s' + 2\{c_r[c_s + c_d k E(\delta^{-1/2})]/\bar{D}'\}^{1/2}$

†$Z^0 \cong E_r[z_0(\lambda, r, \delta)]$

- The statement can also be verified intuitively. If the inbound and outbound schedules can be synchronized, it was shown in "Schedule Coordination" of the 1-to-N systems with transshipments that the inbound and outbound vehicles should arrive and depart from the terminal in perfect synchronization −with every outbound headway a multiple of the inbound headway.

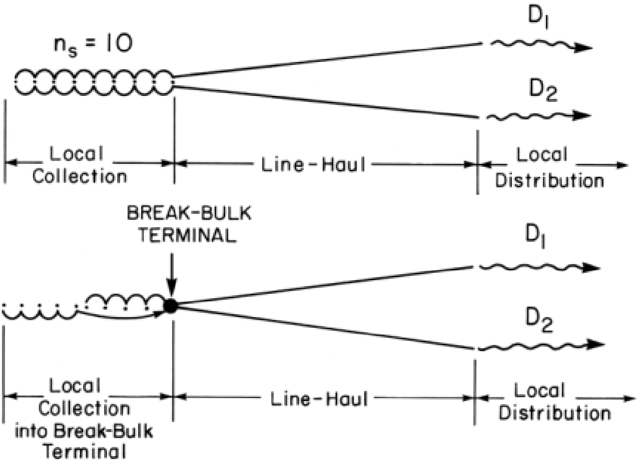
Distribution w/ & w/o transshipments

- The least cost is often obtained with the scheme of Fig.a, where all the headways are equal and there is no discrimination across customers.
- In this case, however, the strategy depicted in Fig.b is also feasible. To see this, recall that delivery vehicles are as large as level-1 vehicles and that there are no route length restrictions. With the same pickup and delivery schedule as the old, the new strategy preserves holding costs and vehicle mileage. This is true because pipeline inventory costs are negligible. Clearly then, the new strategy is more attractive because it saves handling and terminal costs.



Many-to-many multi-commodity problems are different

- Under the same conditions, with no restrictions on delivery vehicle size and route length, transshipments can reduce logistics cost very significantly.
- Suppose that a near optimal solution for a many-to-many problem without transshipments includes two vehicle routes that: (i) visit the same two sets of origins and neighboring destinations, and (ii) are operated with the same frequency.
- This arrangement would be reasonable if the destinations visited are similar



n_s = 10
D_1
D_2
Local Collection
Line-Haul
Local Distribution
BREAK-BULK TERMINAL
D_1
D_2
Local Collection into Break-Bulk Terminal
Line-Haul
Local Distribution

- The top half of the figure depicts the routes that visit the same two sets of origins and neighboring destinations
- The bottom half of the figure illustrates how a transshipment can reduce transportation costs without increasing holding costs at the origins and destinations. Without changing the times of departure, one vehicle could pick up items for both sets of destinations from some of the stops, and the other vehicle would do the same for the remaining stops. Both vehicles would visit just enough stops to carry the same load sizes as in the top part of the figure. To avoid the need for visiting both sets of destinations with both vehicles, these would swap appropriate portions of their loads at a terminal located near the end of their collection runs.
- It should be clear from the figure that such a swap would reduce the distance traveled and the number of stops, without increasing holding costs. It does entail additional fixed, handling and holding costs at the terminal but if the original number of collection stops is large the swap could be cost-effective

- Notice that the magnitude of the savings increases with the number of routes that swap loads at the terminal; e.g., no savings could result with only one origin, or only one destination.
- Clearly, this opportunity for savings is peculiar to many-to-many systems. Because loads must be "broken" before being reconstituted, transshipment points serving this function will be called, consistently with motor carrier jargon, break-bulk terminals (BBTs)

- If there are vehicle size or route length limitations, many-to-many systems may include both consolidation terminals (CTs), whose function is to consolidate the small loads carried by local vehicles into larger (longdistance) vehicle loads, and BBTs serving a swapping function for the CTs.
- This is quite common in existing systems. Motor carrier networks, for example, may include end-of-line terminals (CTs) and break-bulk terminals; railroad networks include industry yards (CTs whose function is accumulating cars from local sidings) and classification yards (BBTs); airline networks include minor airports (CTs) and hubs (BBTs); the postal network is similarly structured, etc.
- For most of this lecture we will focus on the organization of logistic systems with BBTs only, where only one type of vehicle is used. In the meantime CTs will be viewed as the final origins and destinations. We will then show how an integrated system with both CTs and BBTs can be designed.

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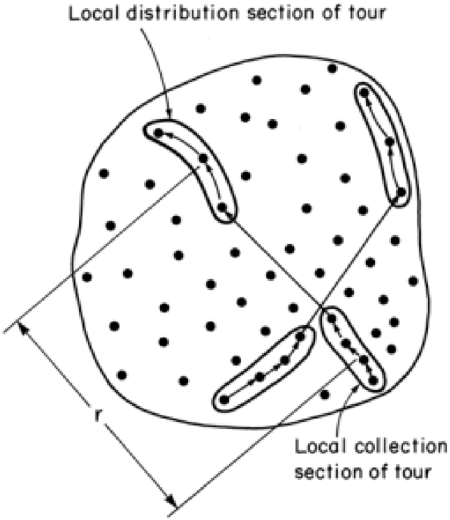
- Let us now examine strategies for serving a collection of scattered origins and destinations without transshipments.
- We will restrict our attention for the time being to transportation modes which are not set up to intermingle（混杂）pick-ups and deliveries within the same route. Not appropriate for passenger transportation, this assumption is reasonable if freight cannot be easily moved within the vehicle.
- Vehicle tours should then stop at only one origin and multiple destinations – or else the other way around. In this manner the freight does not have to be sorted and restowed （重新装载）every time the vehicle stops for a pick-up or delivery.

- Given the spatial densities of origins and destinations $\delta^o(\boldsymbol{x})$ and $\delta^d(\boldsymbol{x})$, and an origin-destination flow density $\lambda(\boldsymbol{x}^o, \boldsymbol{x}^d)$ denoting the number of items per unit time that need transportation from a region of unit area around $\boldsymbol{x}^o$ to a region of unit area around $\boldsymbol{x}^d$ [in this lecture λ has units of items/(time $\times$ distance^4)], we can evaluate the logistics cost per item by comparing 2 strategies:
    1. peddling （行商） with tours from each origin to many destinations,
    2. collecting with tours from many origins to each destination.


Local distribution section of tour
r
Local collection
section of tour

- In the first case, with the transportation, handling and holding cost variables defined as in prior chapters, the cost per item is given by the function $z_0(\lambda, r, \delta) = \text{constant} + \frac{2rc_d}{v_{\max}} + \frac{c_h\delta}{\lambda} v_{\max}$. The arguments "$\lambda, r$, and δ" of this function need to be reinterpreted, though: r now represents the distance between the origin and destinations in a tour (see the figure in the previous slide), δ becomes δ^d, and λ must be replaced by λ/δ^o since in earlier lecture λ represented the number of items demanded per unit time and unit area *from one origin*. Similarly, D' should be replaced by $z_0/(\delta^o\delta^d)$.
- Thus, for peddling, the cost per item averages: $z_0(\lambda/\delta^o, r, \delta^d)$. In the second case, collecting for one destination, the cost is: $z_0(\lambda/\delta^d, r, \delta^o)$; the solution with least cost should be chosen. If desired one can average this minimum over all possible combinations of $\boldsymbol{x}^o$ and $\boldsymbol{x}^d$ to obtain a CA estimate of average cost.

- The explicit cost expressions given in 1-to-N distribution w/ and w/o transshipment, which would be used to develop z_0, assumed that vehicles return to the depot empty after completing the delivery run. For many to many systems, however, this is unlikely; most vehicles will find loads to carry, if not back to the same origins at least somewhere else. With most vehicles usefully employed at their destinations*, one should discount the cost of the return trips.
- It is not difficult to see that the cost of open ended trips can be captured without changing the form of our logistic cost function, e.g. $z = \alpha_0 + \alpha_1(\frac{l}{n_s v}) + \alpha_2(\frac{l}{v}) + \alpha_3(n_s) + \alpha_4(v)$ or $z = \frac{\alpha_1}{A\lambda H} + \frac{(\delta\alpha_2)}{\lambda H} + (\delta\alpha_3)A + c_h H + \alpha_0$, by using r instead of $2r$ in the evaluation of the line-haul transportation cost coefficient α_1; i.e. using $\alpha_1 = c_s + c_d r$. We will assume in the following that α_1 has been adjusted to reflect the availability of backhauls.

*load backhauling will be discussed in the remaining text.

- For a case of peddling cheap items, when pipeline inventory can be ignored, the minimum of the logistics cost function is achieved for $n_s \approx v_{\max}(\alpha_4/\alpha_2)^{1/2}$. The minimum cost is then:

$$z_0 = c_s' + \frac{\alpha_1}{v_{\max}} + 2\alpha_2 \left[\frac{c_h \delta^d \delta^o}{\lambda \alpha_2} \right]^{1/2} .$$

  where we have used $\alpha_4 = c_h \delta^d \delta^o / \lambda$. Recall that α_2 is $c_s + c_d k(\delta^d)^{-1/2}$. It is identical for collecting except for α_2, which is then $\alpha_2 = c_s + c_d k(\delta^o)^{-1/2}$. Clearly, if $\delta^o > \delta^d$ then z_0 is least for collecting, and the reverse is true if $\delta^o < \delta^d$.
- This should be intuitive; it implies that the single stop is made at the end of the trip with the largest traffic generator (either an origin or a destination) and that multiple stops are made at the end of the trip where stops are most closely grouped

*$z = \alpha_0 + \alpha_1(\frac{l}{n_s v}) + \alpha_2(\frac{l}{v}) + \alpha_3(n_s) + \alpha_4(v)$, drop $\alpha_3(n_s)$. $n_s \nearrow, z \searrow$, which leads to $n_s = v_{\max}/v$. Calculate v by the EOQ and then put back to z.

- With our current definitions for α_2 and α_4, the optimal number of stops leading to:

$$n_s = v_{\max}(\frac{\alpha_4}{\alpha_2})^{1/2} = \left\{ \frac{c_h \delta^o \delta^d v_{\max}^2}{\lambda[c_s + c_d k(\delta^d)^{-1/2}]} \right\}^{1/2}.$$

- The right side of this expression is a dimensionless constant that may depend on $\boldsymbol{x}^o$ and $\boldsymbol{x}^d$, and we abbreviate it by $K(\boldsymbol{x}^o, \boldsymbol{x}^d)$. It represents the square root of the ratio of two quantities: (i) **the average load make-up holding cost per item** when every origin-destination pair is served without peddling or collecting by full vehicles, $c_h v_{\max}/D' = c_h v_{\max}\delta^o\delta^d/\lambda$; and (ii) **the prorated motion cost per stop of one item in a full vehicle**, $[c_s + c_d k(\delta^d)^{-1/2}]/v_{\max}$. In other words, K^2 can be viewed as the **ratio of holding cost to transportation cost** for a naive strategy in which one ships in full trucks and allows only one delivery stop; i.e., where only the transportation cost is minimized.

- The quantity "K" can vary by several orders of magnitude depending on the problem at hand and is typically large compared with 1. It can be of order 10^3 (perhaps even larger) when **valuable items have to be moved between many small origins and destinations**, and it is small when the **system consists of few origin-destination pairs with large flows**.
- The constant "K" allows the optimal #. stops and minimum cost to be expressed concisely as follows:

$$n_s = K$$
$$z_o = c_s' + (\alpha_1 + 2\alpha_2 K)/v_{\max}$$

where α_1 is the line-haul motion cost per trip and α_2 is the motion cost per added stop. If one repeats the analysis we have just done, allowing vehicle tours to make both deliveries and pick-ups, one finds that z_0 increases with the 2/3 power of K and $n_s \approx K^{2/3}$.

- Notice that, without transshipments, an unreasonably large number of stops may need to be made. This calls for the introduction of BBTs to shorten vehicle routes. The constant "K" will also appear in the cost expressions for systems with break-bulk transshipments, and will dictate which system configuration is likely to work best. In this lectures, it is assumed that a single vehicle type, with capacity $v_{\max}$, is used. The assumption is relaxed (with the introduction of CT's) in the following lectures

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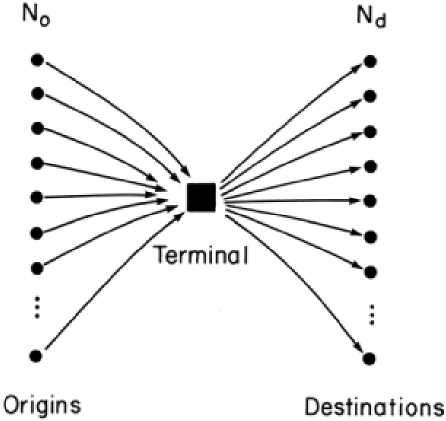
N_o
N_d
Terminal
Origins
Destinations

- The figure shows how by linking N_o origins and N_d destinations through a terminal, the number of two-stop routes is reduced from $N_o \times N_d$ to $N_o + N_d$. The reduction is proportionately larger the larger the number of origins and destinations. This helps reduce transportation cost because, with fewer vehicle routes linking origins and destinations, it is possible to carry the same amount of freight with equal service frequency in larger batches with fewer trips. With larger batches, the transportation cost per item-mile can be reduced by a factor: $(N_o + N_d)/(N_o \times N_d)$.
- Of course, if a smaller reduction in transportation cost is accepted then the service frequency on all the links can be increased

- This route reduction phenomenon is the basis for the one-terminal strategies explored in this section. Symmetric strategies, where origins and destinations are only differentiated by position within the study region $\boldsymbol{R}$, are studied first. With these strategies nearby origins (and nearby destinations) receive similar service.

- We will explore first the operational level (routing) problem where the terminal location and the service frequency (headways) are given; only the item/vehicle routes are sought. Building on this case, we will then examine the tactical level (scheduling/routing) problem where only the terminal location is given. Finally we shall address the strategic level problem.

The operational problem

- With one terminal, the operational level problem is simple. Given a set of inbound and outbound headways — possibly varying across broad subregions of $\boldsymbol{R}$ — the vehicle routes can be found with the solution of a vehicle routing problem, possibly constrained and including backhauls.
- This establishes the distance traveled in $\boldsymbol{R}$ and the total motion cost during each dispatching interval. Since the holding costs are also known, and the number of dispatching instants per day is fixed, the logistics costs per day can be easily estimated.

The tactical problem

- At the tactical level we must choose the schedules and decide whether they are to be coordinated at the terminal or not. Without coordination, the terminal costs per item have the form of $\alpha_5 + \alpha_6/|\boldsymbol{R}|$, which is independent of the tactical variables.

Decoupling the logistics cost

- If the inbound and outbound operations are managed as if they were unrelated many-to-one and one-to-many systems, then the methodology of 1-to-N problems can be used to design each set of operations and to estimate the resulting average inbound and outbound costs.
- In the notation of 1-to-N problems, these costs would be $z_o(\lambda^o, r^o, \delta^o)$, for inbound; and $z_o(\lambda^d, r^d, \delta^d)$ for outbound. Here, r^o (or r^d) represents the distance from an origin (or destination) to the terminal, and λ^o (or λ^d) represents the production (or consumption) rate density in items per unit time per unit area. Clearly, the problem does not require any new treatment.

Schedule coordination

- If schedules can be coordinated, inbound and outbound headways for different subregions of $\boldsymbol{R}$ could be chosen from a menu of the form, $\tilde{H} \times 2^p$, where $\tilde{H}$ is an arbitrary time value and p is an integer. This "power-of-two" strategy allows the average route to use a headway within 50% of optimal, ensuring at the same time that all the headways are integer multiples or submultiples of each other.
- As a result, if all the schedules are forced to coincide at one time (say, at time $t = 0$) then every pair of routes will be synchronized as well as possible and the savings from synchronization will be greatest; they will equal the smaller of the two headways on the two vehicle routes used by any given item (see the integration of the 1-to-N dist. system with production process). In other words, the third term of $(c_i + c_r)\max[H^o; H^i]$ yields the holding cost, which is a convex increasing function of the headways. This facilitates the design process, as explained below.

- Note that the inbound and outbound operations can proceed as described in 1-to-N dist. systems, as if the terminal was the depot. Therefore, the motion cost can be easily estimated. The notation for motion costs introduced in connection with the logistics cost function can be used to summarize these costs.
- There, we denoted by z_m^o the function relating a terminal's outbound motion cost per item to the demand and customer densities, the size of the influence area I, and the headway H. This function also describes the inbound (or outbound) costs in the neighborhood of $\boldsymbol{x}$ for our current situation, if we replace the size of the influence area by $|\boldsymbol{R}|$, λ by the production (or consumption) rate density in the neighborhood of $\boldsymbol{x}, \lambda^o($ or $\lambda^d)$, δ by the density of origins (or destinations), δ^o (or δ^d) , and H by either the origin (or destination) headway H^o (or H^d). Because this function must now be used to describe the sum of the inbound and outbound motion costs, the superscript "*o*" is omitted from the result, which becomes z_m.

*In this lecture, the superscript "*o*" is used to differentiate origins from destinations "*d*", whereas it was used to differentiate the outbound from the inbound direction in 1-to-N dist. system w/ & w/o transshipments

- If backhauls (空载运输) are used, the dependence of the motion costs on the input/decision variables is qualitatively similar but modifications are needed in the expression of z_m. If the flows in both directions are balanced one can simply use r instead of $2r$ in the motion cost expressions of 1-to-N dist. system w/o transshipments, as we suggested earlier.
- If the flows are unbalanced by more than a factor of 2, a good first approximation is to neglect the cost of overcoming distance for the secondary flow and keep all other costs the same. Alternatively, one could use improved formulae such as those proposed in Daganzo and Hall (1990).

- In any case, it is now easy to find headways that minimize approximately the sum of z_m and the holding costs. For example, if the "power of two" constraint is ignored, one can find numerically the headways that minimize the (convex) sum of the holding plus motion costs, averaged over the whole region. The headways can then be adjusted without major repercussions, e.g. to the nearest "power-of-2" multiple of the smallest headway.
- Even more drastically, Daganzo (1990) claims that restricting all the headways to be equal in one of the directions, a helpful simplification, is likely to result in near minimal cost unless there are vast differences among customers.

- A case of special interest arises when vehicles are dispatched full because items are cheap, route length is not restricted, etc.
- Then, for a given set of headways, z_m is given by the minimum of $\alpha_1(\frac{1}{n_s v}) + \alpha_2(\frac{1}{v})$ with respect to n_s; note that the delivery (or collection) lot size v used in the neighborhood of $\boldsymbol{x}$ is fixed since the headway is given.
- Because these terms decrease with the number of stops n_s, the minimum is achieved when $n_s v \approx v_{\max}$.
- Therefore, the inbound part of z_m is:

$$\text{(inbound)} \quad z_m = \frac{\alpha_1}{v_{\max}} + \frac{\alpha_2}{v}.$$

Since $v = (\lambda^o H^o)/\delta^o$ and $\alpha_1 = (c_s + rc_d)$, the inbound motion cost at an origin is:

$$\text{(inbound)} \quad z_m = \frac{c_s + rc_d}{v_{\max}} + \frac{\alpha_2 \delta^o}{\lambda^o H^o}.$$

The destination (outbound) cost expression is identical, with the superscript "o" on the last term replaced by "d".

- Note that the inbound/outbound cost expression increases linearly with r at a rate $c_d/v_{\max}$ that is *independent of the headways*. This should not be surprising, since items travel in full vehicles for any headway. Also note that the expression is convex-decreasing in the headways, as stated. Recognizing that the headways can change with position, we can express the total motion cost per day as:

$$\text{motion cost/day} = \int_{\boldsymbol{R}} [\lambda^o + \lambda^d] \frac{c_s + c_d r}{v_{\max}} \mathrm{d}\boldsymbol{x} + \int_{\boldsymbol{R}} \alpha_2 \left[\frac{\delta^o}{H^o} + \frac{\delta^d}{H^d} \right] \mathrm{d}\boldsymbol{x}$$

- Note that the optimal headways will be the result of a trade-off between the second term of this expression and the holding cost per day. Both elements of the trade-off are independent of r, the distance to the depot. That is, *if the terminal is moved, the optimal schedules do not change.* The only portion of the cost that changes is the first integral of the motion cost, which is a weighted average of r across $\boldsymbol{x}$. This decomposition property will simplify the strategic analysis.

- In other words, if vehicles travel full, the optimal cost of the tactical problem depends on the terminal position (through r) as follows:

$$\text{constant} + \int_{\boldsymbol{R}} [\lambda^o + \lambda^d] \frac{c_d r}{v_{\max}} \mathrm{d}\boldsymbol{x}$$

The same relationship (with a different "constant") holds true without coordination

The strategic problem

- The terminal is optimally located if its distance function $r(\boldsymbol{x})$ minimizes the total tactical costs.
- If vehicles travel full the optimal solution is the minimum of the weighted average expressed by the above integral. This is the well known Weber-point location problem, which can be easily solved*. We reiterate that the optimal location is in this case independent of all other operational and tactical details.

*See Losch, 1954, for example

- If vehicles do not travel nearly full almost always, then the problem does not decompose quite so cleanly. As an approximation, one can calculate the tactical costs for a few candidate locations with different $r(\boldsymbol{x})$, and make the selection accordingly. This approach should be quite satisfactory.
- We have argued repeatedly that logistics problems are usually not very sensitive to the specific dispatching times, terminal locations, etc... if those are reasonably close to the optimum. Numerical experiment with actual data confirms this for the Weber problem.
- S. Bhaskaran and R. Kromer (1986) have done extensive sensitivity analyses for locations of General Motors facilities in the continental US, invariably finding that vast regions of the country provide costs within 1% of optimal. Additional evidence in this respect can be found in Campbell (1992 and 1993a).

- The same should be true for non-Weber problems. For most large systems, one would expect to find substantial portions of $\boldsymbol{R}$ where locating a terminal yields nearly as good a solution as the optimal location. Finding a satisfactory solution, thus, should be easy.
- Hall (1986) has illustrated the process when time-zones are important and there is a deadline for pick-ups and deliveries. Also arguing that the specific location does not matter much, he shows that the ideal region for locating a terminal is shifted eastward due to the asymmetry introduced by the time-zones.

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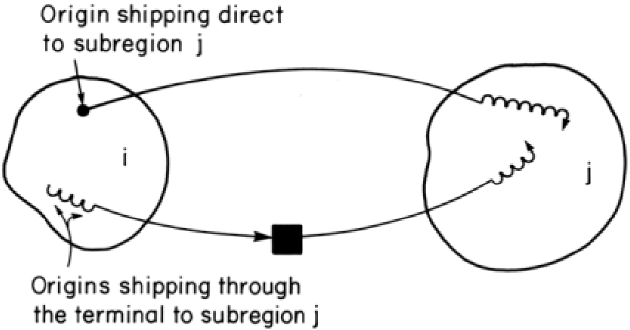
- To this point we have assumed that every origin destination (O-D) pair is served through the terminal. This, however, may be inefficient when the origin is close to the destination and both are far from the depot. Too much circuity is introduced.
- Using a conditional decomposition procedure similar to that of discriminating strategies for 1-to-N problem w/ transshipment, we examine in this subsection ways of discriminating across O-D pairs. We examine first strategies that only differentiate O-D pairs by their general location within R, and then discuss briefly more detailed strategies that account for other characteristics of the origins and destinations.

Decomposition by location

- Instead of dealing with O-D pairs individually, they will be treated in groups. To accomplish this, the service region is divided into origin subregions i and destination subregions j (hopefully not too many); O-D pairs for the same two subregions (i, j) are then treated in the same manner. We use P_i and P_j to denote the surface areas of these subregions. In the following, variables indexed by superscript "i" will refer to origin zones, and variables indexed by "j" to destination zones.


Origin shipping direct
to subregion j
i
j
Origins shipping through
the terminal to subregion j

- If the inbound and outbound headways at the terminal are known for every i and j, then the operational problem is simple. We need to determine what proportion of the flow from subregion i to subregion j, f^{ij}, should travel direct, and what proportion through the terminal (figure in previous slide).
- If for direct distribution between subregions i and j it is better to peddle than collect, we will assume that fij of the origins in subregion i ship to j on a direct, peddling route, and the remaining origins through the terminal. The desired split is then achieved by partitioning the origin subregion into a direct-shipping and an indirect-shipping part. (The same flow split between i and j can be achieved in other ways, but the cost can be shown to be higher).
- The average cost for items shipped direct, z^{ij} could calculate by $z_0 = c_s' + \frac{\alpha_1}{v_{\max}} + 2\alpha_2[\frac{c_h\delta^d\delta^o}{\lambda\alpha_2}]^{1/2}$, if vehicles travel full. Note that z^{ij} is a constant independent of all tactical and operational variables. Most notably, it is independent of the f^{ij}. We now show that under certain conditions, the operational problem decomposes by O-D subregion pair

- Conditional on the headways Hi and Hj , the holding cost per item for travel from i to j through the terminal is known and independent of f^{ij}. Recall that it equals $c_h \max\{H^i, H^j\}$ if the schedules are perfectly coordinated.
- The motion costs per item (z^i_m and z^j_m) are also as described in symmetric strategies, but with smaller origin and destination flow densities:

$$\begin{cases} \lambda^i & = \sum_i [\lambda^{ij}(1 - f^{ij}) P^i], \quad \text{for production} \\ \lambda^j & = \sum_j [\lambda^{ij}(1 - f^{ij}) P^j], \quad \text{for consumption} \end{cases}$$

- The total terminal-motion cost per day from i to the terminal is thus: $\lambda^i P^i z^i_m$. A similar expression holds for the destinations j. Thus, the total terminal-motion cost per unit time is:

$$\sum_i \lambda^i P^i [z^i_m] + \sum_j \lambda^j P^j [z^j_m]$$

- If vehicles are dispatched full, we have:

$$z^i_m = \frac{(c_s + r^i c_d)}{v_{\max}} + \frac{\alpha_2 \delta^i}{\lambda^i H^i},$$

and the total terminal-motion cost per day is:

$$\sum_i \left[\frac{\lambda^i (c_s + r^i c_d)}{v_{\max}} + \frac{\alpha_2 \delta^i}{H^i} \right] P^i + \sum_j \left[\frac{\lambda^j (c_s + r^j c_d)}{v_{\max}} + \frac{\alpha_2 \delta^j}{H^j} \right] P^j$$

- We have used the constants δ^i and δ^j (densities of origins and destinations) in these expressions for the density of collection and delivery stops to/from the terminal. This is reasonable for the origins, in view of the partitioning scheme for splitting the flows, and is also reasonable for the destinations as long as every destination receives some flow through the terminal.
- Therefore, it is safe to assume that in the expression for the total terminal-motion cost per day only the λ^i and λ^j depend on the splits and that, as explained earlier, the dependence is linear. Note as well that the rate at which cost increases with the splits is independent of the headways, as happened in the symmetric strategies.

Logistic cost function

If the headways are constant the terminal-motion cost per day is the sum of a constant plus an amount $(c_s + rc_d)/v_{\max}$ for every item collected and every item delivered r miles away from the terminal. The contribution of O-D subregion pair (i, j) toward this quantity is:

$$\frac{\lambda^{ij} P^i P^j (1 - f^{ij})(2c_s + \lfloor r^i + r^j \rfloor c_d)}{v_{\max}};$$

its contribution toward the daily holding-terminal costs is:

$$\lambda^{ij} P^i P^j (1 - f^{ij})[c_h \max(H^i, H^j)]$$

and its contribution toward total daily direct-shipping costs is:

$$\lambda^{ij} P^i P^j f^{ij} z^{ij}$$

The sum of these three expressions across (i, j) is a logistic cost function, which is to be minimized with respect to the f^{ij} for a given set of headways (the operational problem).

- The f^{ij}'s must be in the unit interval; they are not restricted by other constraints. Therefore, since the objective function is separable in the f^{ij}, each f^{ij} can be chosen independently of the others, by minimizing its contribution to the objective function. In physical terms this means that for a given set of headways, we will ship without transshipments if the direct-cost per item, z^{ij}, is smaller than the marginal cost of sending an item through the terminal:

$$\frac{2c_s + [r^i + r^j]c_d}{v_{\max}} + c_h \max(H^i, H^j).$$

- Otherwise, we should ship through the terminal. Because f^{ij} should be either 0 or 1, all the sources in an origin subregion will ship in the same manner.

- If the system is operated on the clock with a unique headway H, and the conditions are fairly homogeneous — so that z^{ij} is the sum of a constant and $r^{ij}c_d/v_{\max}$, where r^{ij} is the distance from i to j — then the decision for f^{ij} is only based on the circuity of the terminal route. We ship direct whenever $r^i + r^j - r^{ij}$ is greater than a fixed quantity.
- If the vehicles cannot be operated full, then the operational problem is more complicated because the f^{ij}'s don't define a separable linear program anymore, but it should be possible to solve it approximately with relaxation schemes. Note, however, that with the high flows likely to arise in this type of problem (otherwise we would not be considering the direct shipping option to begin with) it is quite unlikely that route length constraints or pipeline inventory cost considerations would prevent filling the vehicles.

- The tactical problem is easy to solve if the operational problem can be solved. If the system is operated on the clock (with a unique H), it is a simple matter to choose the best H by testing various values; the one with the least cost should be chosen.
- If a unique headway is not desirable the solution is more difficult. We have seen, however, that conditions have to be drastically different for some i's and j's for that to be the case. It should then be fairly obvious which (i, j)'s should be operated on lower or higher frequencies than the norm; and to identify a few reasonable headway sets for testing.

More detailed discrimination

- Decomposition approaches also work if other details of the origins and destinations, besides location, are also used to discriminate across O-D pairs. This is desirable if the additional discriminating characteristics, such as production rates and item values, change drastically across origins and destinations.
- In this case, instead of partitioning the region into subregions, one must deal individually with the specific origins and destinations; but this detailed approach is not difficult to apply if vehicles are not allowed to either peddle or collect.

- For this scenario, Hall (1987) bases routing decisions on both location and the specific demand and production rates. He assumes that the total logistic cost of the links inbound to the terminal is linear in flow. (We have seen already that this will happen if these links are operated with full vehicles; thus the assumption implicitly assumes that the flows on these links are fairly substantial).
- With this assumption, the problem decomposes by destination: the best way of receiving items at each j from all the i's can be determined independently for all j's. Each of these destination subproblems can be further decomposed if one holds constant the headway (or the shipment size as Hall suggests) from the terminal to the destination. The method can be applied with and without coordination at the terminal; and can probably be extended to allow for multi-stop non-terminal routes.

- Blumenfeld et. al (1985a) have addressed the same problem without the linearity condition, but their method only works for few (3 or 4) origins and any number of destinations. The problem is also decomposed by destination; but to achieve independence one must condition jointly on the inbound headways (or shipment sizes) from every origin to the terminal.
- It is impossible to explain here all the possible situations and how they could be addressed (see Hall, 1993b, for more examples). Suffice it to say that if some sort of asymmetric service is suspected to be beneficial it might be possible to use a decomposition method if a proper set of conditioning (tactical) variables can be found.

Any questions?

Readings

- Daganzo. Logistics System Analysis. Ch.6. Page 215-232.