物流系统分析
Logistics System Analysis

第 15 周　带转运的一到多配送问题（3）—多次转运与自动分区
One-to-Many Distribution with Transshipments — Multiple Transshipments and Beyond

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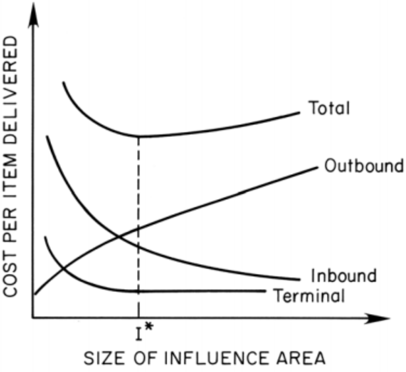
Reasons for Using Terminals

- Items are often transshipped when there is an incentive to change transportation modes or vehicle types.
- While **geographical barriers** such as coastlines invariably require a modal change (e.g. at seaports), purely **economical considerations** may also encourage changes in vehicle type.
- We may find the **optimal dispatching frequency** and **the size of the influence area** l^* for the system

The Design Problem



The total cost per item distributed is the sum of the terminal, inbound and outbound costs:

$$\underbrace{\alpha_5 + \alpha_6/I}_{\text{terminal}} + \underbrace{z^i(\lambda, r, I)}_{\text{inbound}} + \underbrace{Z^0(\lambda, \delta, I)}_{\text{outbound}}$$

入站成本不依赖于 δ，因为此时的“顾客点”为中转枢纽，其密度与影响区域的大小成反比，$\propto I^{-1}$；
出站成本不依赖于 r，因为服务区域可视为一个圆，每个顾客点到中转枢纽的期望距离是 $2R/3 = 2/3 \times \sqrt{I/\pi} \approx 0.38I^{1/2}$

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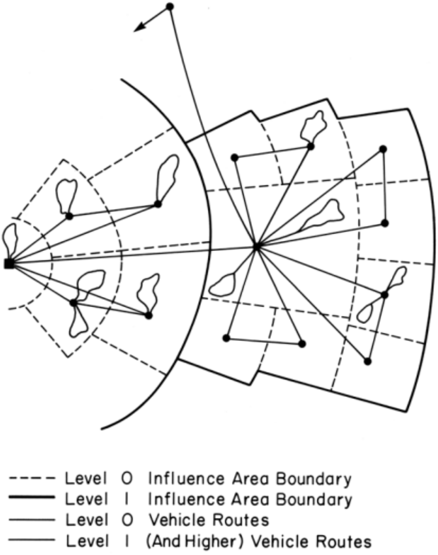
View of the Level-I Influence Area

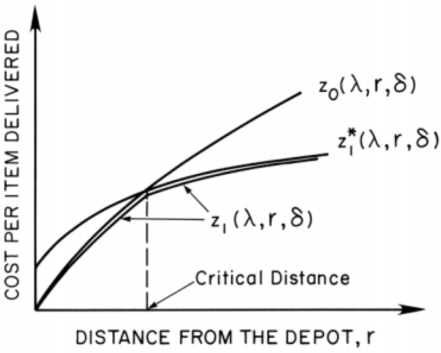


View of the Level-I Influence Area (cont.)

- The figure depicts a level-1 terminal and its influence area, whose size is now denoted $I_1(\boldsymbol{x})$. Recall that all the customers in a level-1 area are served from the level-1 terminal with at most 1 transshipment, not including the one at the level-1 terminal, and that the level-1 terminals themselves are served without transshipments from the depot.
- This structural organization makes it easy to express, conditional on I_1, the inbound, outbound and terminal costs for a level-1 terminal; the logistic cost function is now:

$$\text{cost/item} = \underbrace{(\alpha_5 + \alpha_6/I_1)}_{\text{terminal cost}} + \underbrace{z^i(\lambda, r, I_1)}_{\text{inbound cost}} + \underbrace{Z^1(\lambda, \delta, I_1)}_{\text{outbound cost}}$$

成本分析



- The terminal and inbound costs assume the same functional form as in the expression for distribution systems with one terminal, since the cost of delivering and passing through the level-1 terminals does not depend on how the items are treated once they leave them.
- The outbound cost is superscripted by "1" since Z_1 should now represent the average of $z_1(\lambda, r, \delta)$ instead of the (larger) $z_0(\lambda, r, \delta)$

不建议多次转运

- Multiple transshipments are unlikely to be advisable for most physical distribution applications, because **each additional transshipment generates additional handling costs** and **the vehicle economies** (v_{max} vs. v'_{max}) **can be achieved with just one transshipment**.
- In any case, systems that allow multiple transshipments can be designed, using the one-transshipment results as a building block.

This lecture presents a simple recursive technique to this effect, and illustrates it with an example. The technique uses the function $z_1(\lambda, r, \delta)$ to construct a function $z_2(\lambda, r, \delta)$ representing the minimum cost per item with at most two transshipments.

Outbound cost of the level-1 influence area

- We may want to approximate the average cost by the cost of the average:

$$Z^1(\lambda, \delta, I) \cong z_1(\lambda, 0.38 I^{1/2}, \delta)$$

  but the accuracy of this approximation will now have deteriorated because z_1 is more highly non-linear as a function of r than z_0.

- One may instead opt for using the exact definition:

$$Z^1(\lambda, \delta, I) = E_r[z_1(\lambda, \delta, r)]$$

- Either one of these expressions can be used to find the minimum of the logistics cost with respect to I_1. The result should be a function of λ, r, and δ.

- $z_2^*(\lambda, r, \delta)$, as a function of r, should start higher and be flatter than either z_0 or z_1^*. 使用二级转运起始成本可能较大，但是相比无转运或者仅一次转运，曲线更“平”。
- As a result, we may find a second critical distance beyond which two transshipments are needed ($z_2^* < z_1$). For most practical problems, this distance is likely to be large compared with the distance between the depot and the farthest reaches of $\boldsymbol{R}$.

- It is theoretically possible, but practically unnecessary, to iterate this procedure to obtain the optimal size of higher level influence areas. The technique can also be applied if shipments are to be synchronized at the level-1 terminals and also if constraints require a more extensive list of conditioning variables for the decomposition principle to apply.
- In this case one would minimize $z^i_m(\lambda, r, I, H^i) + z^o_m(\lambda, \delta, I, H^o) + (c_r + c_h)\max[H^o; H^i$ $(\alpha_5 + \alpha_6 I^{-1})$ holding H_i constant, and this variable would appear in the expression for Z_1. The new expression would then include the inbound and outbound headways as decision variables, in addition to I_1

- In order to design the system one would carve out the service region into influence areas approximating the ideal size $I_1(\boldsymbol{x})$. Of course, this only needs to be done for the portion of $\boldsymbol{R}$ lying beyond the second critical distance.
- The headways at the level-1 terminals, a byproduct of the optimization, can be used to construct the level-1 feeder routes and schedules.
- Within each level-1 influence area, the system can be designed as previously. An example illustrates the procedure.

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- The example that led to z_1^* in the primal one transshipment problem is continued here.
- To simplify the notation we will give some arbitrary values to the constants that appeared: $v_{\max} = b = b' = a/\lambda = a'/\lambda = 1$, and will then eliminate these variables from the notation. We assume that the demand and customer density do not depend on location or time, and use the case with $v'_{\max} = \infty$.

一次转运的计算

Recall the cost expressions:

$$z_0(\lambda, r, \delta) = \frac{av_{\max}}{\lambda} + [\frac{2.7b}{v_{\max}}]r = 1 + 2.7r$$

$$Z^0(\lambda, \delta, I) \cong \frac{av_{\max}}{\lambda} + \frac{b}{v_{\max}} I^{1/2} = 1 + I_0^{1/2}$$

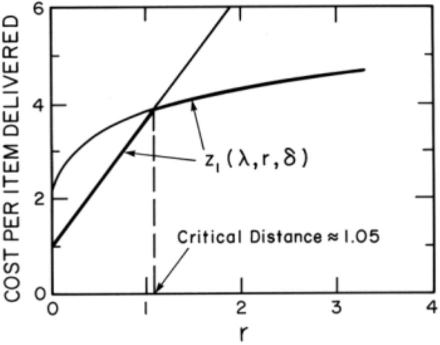
$$z^i(\lambda, r, I) = 2\left(\frac{a'b'r}{\lambda I}\right)^{1/2} = 2(r/I_0)^{1/2}$$

$$I_0^* \cong \left(\frac{2v_{\max}}{b}\right)\left(\frac{a'b'r}{\lambda}\right)^{1/2} = 2r^{1/2}$$

$$z_1^* \cong a\frac{v_{\max}}{\lambda} + 2.83\left(\frac{b}{v_{\max}}\right)^{1/2}\left(\frac{a'b'r}{\lambda}\right)^{1/4} = 1 + 2.8r^{1/4}$$

Putting in the given values, we know

$$z_0 = 1 + 2.7r,$$
$$Z^0 = 1 + I_0^{1/2},$$
$$z^i(r, I_0) = 2(r/I_0)^{1/2},$$
$$I_0^* = 2r^{1/2},$$
$$z_1^* = 1 + 2.8r^{1/4}.$$



Thus

$$z_1(r) = 1 + \min\{2.7r, 2.8r^{1/4}\}$$

Note that when $r \geqslant 1.05$, transshipments become necessary.

To calculate $Z^1(I_1)$, one should take the expectation of $z_1(r)$ for the r values that arise in an influence area of size $I_1 : r \in [0, (I_1/\pi)^{1/2}]$.

- For small influence areas $(I_1 \leqslant 0.29^{-1})$, $z_1(r) = 1 + 2.7r$ and

$$Z^1(I_1) = 1 + I_1^{1/2}, \text{ if } I_1 \leqslant 0.29^{-1}.$$

  In this case, the level-1 influence area is not large enough to require another transshipment.

- For $I_1 \geqslant 0.29^{-1}$, we find

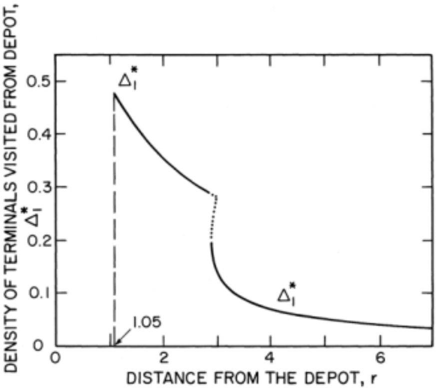
$$\begin{aligned} Z^1(I_1) &= E_r[z_1(\lambda, r, \delta)] \\ Z^1(I_1) &= 1 + 2.7\int_0^{1.05} \frac{r \times 2\pi r}{(I_1/\pi)^{1/2}} \mathrm{d}r + 2.8\int_{1.05}^{(I_1/\pi)^{1/2}} \frac{r^{1/4} \times 2\pi r}{(I_1/\pi)^{1/2}} \mathrm{d}r \\ &= 1 + 2.17(I_1^{1/8} - I_1^{-1}) \text{ if } I_1 \geqslant 0.29^{-1}. \end{aligned}$$

二次转运总成本的计算

Applying the optimal l_1^*, cost per item now becomes (remember that we assumed $\alpha_5 = \alpha_6 = 0$):

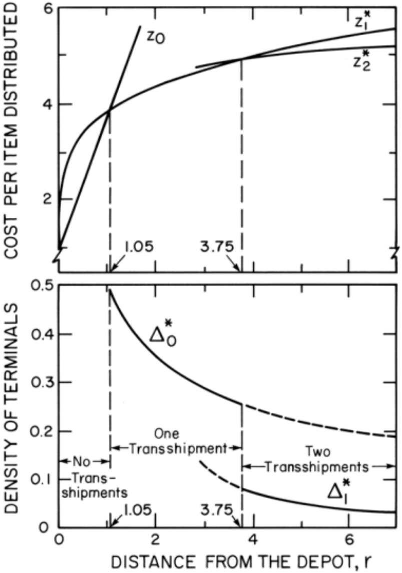
$$z_2^* = \begin{cases} 2(r/l_1)^{1/2} + 1 + l_1^{1/2} & \text{if } l_1 < 0.29^{-1} \\ 2(r/l_1)^{1/2} + 1 + 2.17(l_1^{1/8} - l_1^{-1}) & \text{if } l_1 \geqslant 0.29^{-1}. \end{cases}$$

- This expression should now be minimized for all values of r. For this particular problem the task is easy. One can find for every l_1^*, the value of r that makes it optimal — and one can be plotted against the other.
- The right figure plots the reciprocal (关联) of l_1^* as a function of r.



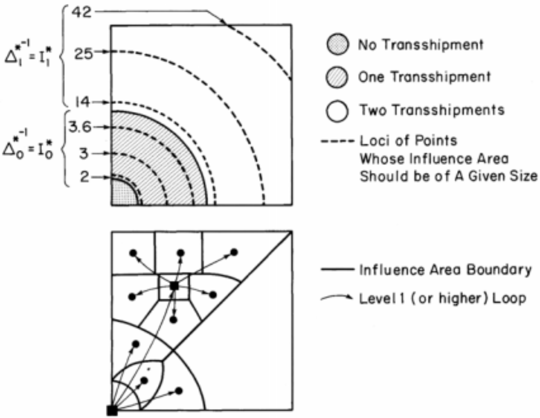
Optimal density of terminals as a function of distance

- The right figure plots the minimum cost as a function of r as well.
- When r reaches 3.75, the cost, z_2^*, equals z_1. For larger values, two terminal shipping is best



Partition of service zone into influence areas

- This figure depicts a possible configuration of influence areas for a square of side that attempts to be true to the density of terminals shown in the previous page
- For this particular problem the task is easy. One can find for every I_1^*, the value of r that makes it optimal — and one can be plotted against the other.



Partition of service zone into influence areas (cont.)

- Unfortunately, the size of the influence areas forces them to include points that would be better served with larger or smaller influence areas. For example, the level-1 influence zones have an area of approximately 20 units, but they include points that optimally would require $I_1 = 13$ to $I_1 = 42$, plus a few corners with even more stringent requirements.
- Inspection of the expression for cost per item, reveals that variations from the optimal I_1 by a factor of 2 only increase the objective function by about 1%.
- This robustness is even more pronounced than that observed for level-0 influence areas because the exponents of the objective function are now closer to zero

$\Rightarrow$ the departures from optimality observed in the previous page should not matter much.

Fine-tuning

- The exact location of the boundaries and terminals can be fine tuned if desired, but since they are fairly round and centered, respectively, the configuration shown should be nearly optimal.
- In fact, even the precise location of the boundary between 2 and 1 transshipment service areas is not particularly crucial. The following section describes an automatic way to fine-tune, or even develop a design.

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- Before starting, we should mention that the design problem has also been treated in the literature as a pure optimization exercise - without resorting to the CA approach. In the applied mathematics literature the problem is called the "**optimal resource allocation problem**"*
- Pertinent works seek cost-minimizing locations for **point-like service facilities in a space continuum**, among a continuum of customers. Unfortunately, these optimization problems turn out to be "easy" only when cost is defined as a simple function of a distance norm.
- This cost structure, e.g., with the translational symmetry implied by a norm, is unrealistic for typical logistics problems where costs are complicated and almost invariably location-dependent.

*see Okabe et al. (1992) and Du et al. (1999)

选址问题 (cont.)

- More realistic cost scenarios can be analyzed by considering discrete versions of the problem with only a finite number of locations*.
- Problems of this type are usually solved with mixed-integer programming techniques, where the terminal locations and customer allocations are decision variables.
- But unfortunately, existing programming methods can only deal effectively with small problems if they have complicated cost structures.

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*An extensive operations research literature explores this line of inquiry; see e.g., Daskin (1995) and Drezner and Hamacher (2002).

CA 的选址方法

- The manual method overcomes these drawbacks. It succeeds, e.g., as in the example of multiple transshipments because it decomposes the problem in two manageable parts.
- We first look for a continuous target $I^*(\boldsymbol{x})$ without paying attention to the discrete locations, and then delegate the difficult but non-crucial task of finding the specific locations to the human mind. As explained in the design problem, the human designer is simply asked to partition the service region into "round" influence areas $\{I_i\}$ of a size consistent with the CA target $I^*(\boldsymbol{x})$, and a set of centrally-located terminals $\{\boldsymbol{x}_i\}$.
- The remainder of this lecture* shows that this second step can also be performed automatically, even for large problems.

*One may refer to Ouyang and Daganzo (2004) for details

自动分区算法

- Because roundness is important, we first look for a set of nonoverlapping circular disks contained within the service region, of individual sizes as close the ideal $I^*(\boldsymbol{x})$ as possible.
- The number of disks is given by the CA procedure: $N = \int I^*(\boldsymbol{x})^{-1} \mathrm{d}\boldsymbol{x}$. More specifically, if we characterize the disks by their centers $\boldsymbol{x}_i$ and their radii r_i (for $i = 1, 2, \ldots N$), we look for a set of $(\boldsymbol{x}_i, r_i)$ that satisfy: $I^*(\boldsymbol{x}_i) \approx k\pi r_i^2$ for $i = 1, \ldots N$, for a value of k as close to 1 as possible.
- Once this is done, we generate influence areas by allocating each point in the service region to the nearest $\boldsymbol{x}_i$. This is the right thing to do because it guarantees that the influence areas so generated contain one disk a piece. Therefore, they must be "round" - assuming that a solution with $k \approx 1$ has been found.

自动分区算法 (cont.)

- To find a set of disks, we assign some initial values to the $(\mathbf{x}_i, r_i)$ and model the disks as if they were physical particles that (i) are repelled when they overlap either with each other or with the boundary, and (ii) change radius as they move over the service region with the recipe: $r_i \approx [I^*(\mathbf{x}_i)/k\pi]^{1/2}$. If k is sufficiently large, a discrete-time simulation of this system quickly leads to an equilibrium where all forces vanish and there is no overlap*.
- The simulation is then repeated with a smaller k. A step-wise gradual reduction in k is continued until an equilibrium cannot be found. This will happen before $k = 1$, since circles do not partition Euclidean space. The procedure is then terminated.
- This procedure can quickly find good designs to problems of practical size†.

*This assumes that the service region is "simply connected", in the sense that a disk of proper size can always be slid between any two points in the service region without touching the boundary. No generality is lost by this assumption, because complex areas (e.g., Japan) can usually be partitioned into simply connected components to which the model can be applied separately

†reported in Ouyang and Daganzo (2004)

Algorithm 1: 自动分区算法

$m \leftarrow 1$; 初始化中转枢纽位置 $\boldsymbol{x}_i$ 和影响区域半径 r_i; 预设参数：收敛容许参数 ϵ，步长 μ_m，扰动参数 δ，设置影响区域的面积系数 $k \approxeq 1$，面积系数的变动参数 Δk

while $F_T \neq 0$ 或 $F_B \neq 0$ **do**

  计算每个 disk 的大小：$r_i = \sqrt{\frac{I^*(\boldsymbol{x}_i)}{k\pi}}$

  计算重叠 disk 的斥力 F_T（依赖于 $r(\boldsymbol{x}_i) + r(\boldsymbol{x}_j)$ 与 $||\boldsymbol{x}_i - \boldsymbol{x}_j||$ 之间的关系）和边界的斥力 F_T（依赖于 $r(\boldsymbol{x}_i)$ 与中转枢纽到边界距离的关系）

  if $F_T \neq 0$ 或 $F_B \neq 0$ **then**

    将中转枢纽沿着斥力方向移动 μ_m，同时随机在某个方向增加一个扰动 δ

  end

  if $\mu_m < \epsilon$ **then**

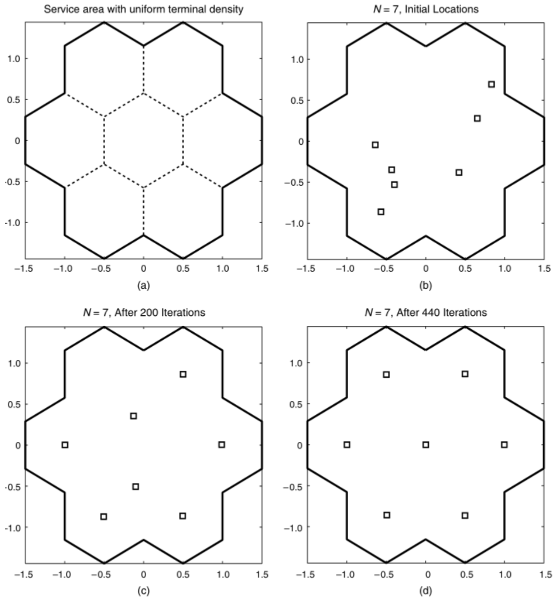
    重置 $m = 1$，$k = k + \Delta k$

  end

  $m = m + 1$

end

基于带权重的 Voronoi 曲面细分方法（weighted–Voronoi tessellation，WVT），划分中转枢纽的影响区域，$i = \arg\min_j \frac{||\boldsymbol{x} - \boldsymbol{x}_j||}{r(\boldsymbol{x}_j)}$。

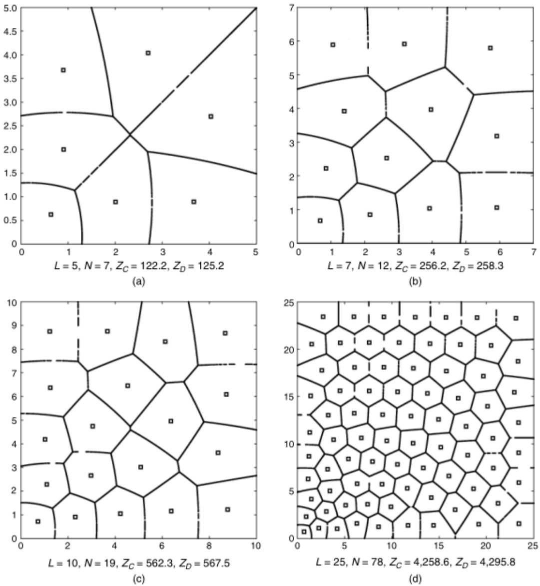


Service area with uniform terminal density
1.0
0.5
0
-0.5
-1.0
–1.5
–1.0
–0.5
0
0.5
1.0
1.5
(a)
N = 7, Initial Locations
(b)
N = 7, After 200 Iterations
(c)
N = 7, After 440 Iterations
(d)

- The figure shows how the method converges in a case where the best design is known. The region is poly-hexagonal with $N = 7$, and the target area size $l^*(\boldsymbol{x})$ is independent of location. The best design is shown in (a), and (b)-(d) show the production of the algorithms of (1) initial locations, (2) location after 200 iterations, (3) equilibrium after 440 iterations respectively.

- The algorithm has also been applied to the example in lecture on the one transshipment problem using $l^* \cong \frac{2v_{\max}}{b} \times (\frac{a'b'r}{\lambda})^{1/2}$ as the target function with $a = b = a' = b' = v_{\max} = 1$; i.e. $l^*(\boldsymbol{x}) = 2[r(\boldsymbol{x})/\lambda(\boldsymbol{x})]^{1/2}$. (Recall that $r(\boldsymbol{x})$ was the Euclidean distance to the depot, and $\lambda(\boldsymbol{x})$ the demand density.)
- Two cases were considered: (a) uniform demand, where $\lambda = 1$ and $l^*(\boldsymbol{x}) = 2r(\boldsymbol{x})^{1/2}$; and (b) declining demand, where $\lambda(\boldsymbol{x}) = r(\boldsymbol{x})^{-1/2}$ and $l^*(\boldsymbol{x}) = 2r(\boldsymbol{x})^{3/4}$. The following two slides show the results for four square regions of sides $L = 5, 7, 10$ and 25 when the customer demand is homogeneous and inhomogeneous, respectively.

Solution for homogeneous customer demand



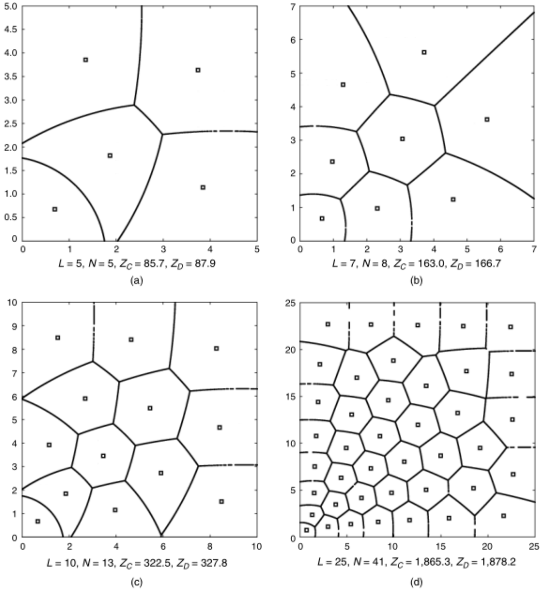
$L = 5$, $N = 7$, $Z_C = 122.2$, $Z_D = 125.2$
(a)

$L = 7$, $N = 12$, $Z_C = 256.2$, $Z_D = 258.3$
(b)

$L = 10$, $N = 19$, $Z_C = 562.3$, $Z_D = 567.5$
(c)

$L = 25$, $N = 78$, $Z_C = 4{,}258.6$, $Z_D = 4{,}295.8$
(d)

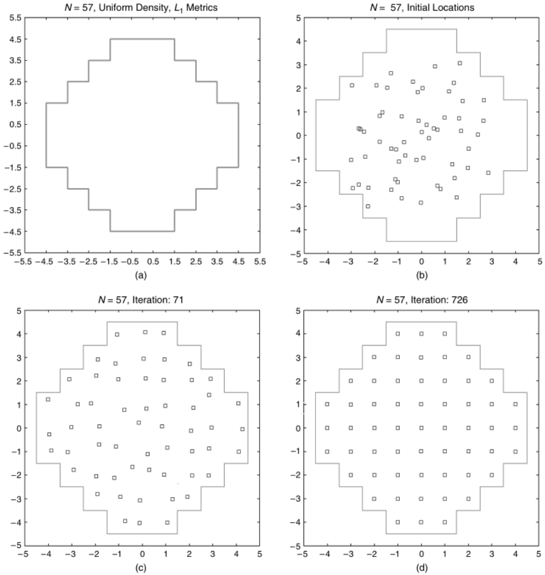
Solution for inhomogeneous customer demand



- In the uniform demand case the difference between the CA cost prediction for the variable costs −the integral $2.83(\frac{b}{v_{max}})^{1/2}(\frac{a'b'r}{\lambda})^{1/4}$ over the service region — and the variable costs arising from the design is quite small: 2.4% for $L = 5$, 0.8% for $L = 7$, 0.9% for $L = 10$, and 0.9% for $L = 25$.
- In the variable demand case the cost differences are 2.6%, 2.3%, 1.6%, and 0.7% respectively. All these differences are exaggerations because they ignore fixed costs, such as $a\frac{v_{max}}{\lambda}$, which are large and can be predicted without error by the CA method.
- In all cases, the CA prediction was lower than the actual cost. This is not a coincidence. The CA predictions for our examples should be lower bounds to the optimum solution. Thus, the percentage differences we observed can be interpreted as optimality gaps.

- Note that in both scenarios, and in agreement with theory, the accuracy of the CA formulae and the efficiency of the proposed design method improves with problem size considerably.
- It means both, that the CA formulae describe well the optimum costs of large complex problems, and that the CA discretization algorithm can complement conventional optimization methods when they would have the most difficulty.

Although the discretization procedure was illustrated with Euclidean metrics, it can also be applied to other metrics by deforming the disks during the simulations, and using true distances in the tessellation step. For example, designs for L_1 metrics should use square “disks” with the same repulsive forces as before, and the L_1 distance formula. An example is shown in next slide.



N = 57, Uniform Density, L_1 Metrics
(a)
N = 57, Initial Locations
(b)
N = 57, Iteration: 71
(c)
N = 57, Iteration: 726
(d)

Any questions?

Readings

- Daganzo. Logistics System Analysis. Ch.5. Page 195-206.