物流系统分析
Logistics System Analysis

第 8 周　一到多配送问题（1）—非详细的车辆路径问题
One-to-Many Distribution – Non-detailed Vehicle Routing Problems

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目 录 |Outline

- We plan several lectures to address the physical distribution problems where items **produced at a single origin** are to be taken, **without transshipment**, **to a set of scattered destinations** over a service region $\boldsymbol{R}$.
- For the most part, we will focus on **delivery problems**, although it should be recognized that **collection problems** from many sources to a single destination are mathematically analogous.
- The objective is to obtain simple guidelines for the design of a set of **routes and delivery schedules** that will **minimize the total cost per unit time**.
- The CA approach for the 1-1 problem will be extended to the 1-N problem; yielding in the process simple formulae for the total resulting cost.

- The continuum approximation method is most accurate for one-dimensional point location problems if **the characteristics of the problem vary slowly along the location domain** (e.g., the time or distance line).
- The current problem is much more complex. In addition to a schedule for every customer, we must design a set of time varying routes to meet the schedule.
- It can be reduced to a point location problem in multiple (time-space) dimensions; accordingly, our solutions will be most accurate if the characteristics of the problem vary slowly over both space and time. → 本节中所涉及的问题可以约减为一个多维（时间 + 空间）中的单点选址问题。因而，只有当时空特征均缓慢变化时，所给出的解最准确。

The scenario

- A large number of destinations/customers N is distributed over a region $\boldsymbol{R}$
- The density function is a slow varying continuous function $f(\boldsymbol{x})$ of the point coordinates $\boldsymbol{x} = (x_1, x_2) \in \boldsymbol{R}$

That is, the actual number of points in a subregion of $\boldsymbol{A} \subseteq \boldsymbol{R}$, is approximately given by:

$$\int_{\boldsymbol{x} \in \boldsymbol{A}} \mathrm{Nf}(\boldsymbol{x}) \mathrm{d}\boldsymbol{x}.$$

If $f(\boldsymbol{x})$ remains nearly constant over a small $\boldsymbol{A}$, then the number can be written as:

$$\mathrm{N} \int_{\boldsymbol{x} \in \boldsymbol{A}} \mathrm{f}(\boldsymbol{x}) \mathrm{d}\boldsymbol{x} \approx N\mathrm{f}(\boldsymbol{x}_a)|\boldsymbol{A}|.$$

where $\boldsymbol{x}_a$ is any point in $\boldsymbol{A}$, and $|\boldsymbol{A}|$ is the area of $\boldsymbol{A}$.
Note that a design approach based on expressions of this type can be used even before the actual point locations are known.

The scenario (cont.)

- In the literature, a common interpretation of $f(\boldsymbol{x})$ is as a probability density function for the coordinates of the customers, assumed to be located independently of one another.
- In that case, the above expressions represent the mean number of customers found in subareas of $\boldsymbol{R}$;
- The actual number can vary across subareas with the same mean. The standard deviation (SD) is $\{\mathrm{N}f(\boldsymbol{x}_0)|\boldsymbol{A}|[1-f(\boldsymbol{x}_0)|\boldsymbol{A}|]\}^{1/2*}$ when f(x) is nearly constant over $\boldsymbol{A}$ and points are located independently.
- The SD grows with N and $\boldsymbol{A}$ more slowly than the mean. These variations do not prevent continuous approximations to improve as N grows.
- Assume also that the cumulative number of items demanded by each customer can be expressed as a demand curve $D_n(t)(n = 1, 2, \ldots, N)$, which is assumed to vary slowly with t

*任意顾客落在区域 $\boldsymbol{A}$ 中的概率为 $|\boldsymbol{A}|$，因此区域 A 中顾客数服从二项分布。

Topics

Each topic may take one lecture.

- **Non-detailed** vehicle routing problem
- Customers with **homogeneous demand**: vehicles are filled to capacity at the depot; vehicles are not filled to capacity at the depot; detailed solution from the guidelines
- Customers with **heterogeneous demand**: symmetric strategies extended from previous discussions for the case that customers are identical; asymmetric strategies
- **Integration with production process**: adjustable production process without penalty; relaxation to handle general problems

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Non-detailed Vehicle Routing Models

- Given a set of delivery schedules for the customers in the region, one should use the vehicle routes which **minimize total distance traveled**. The toal travel distance the main determinant of transportation cost.
- It is assumed that items are distributed with identical vehicles capable of carrying $v_{\max}$ items. This definition of vehicle capacity can be used even if different item types move through the system, simply by redefining the concept of "item".
- If the maximum freight volume (or weight) that can be carried by a vehicle does not depend considerably on the mixture of item types making up its load, one can think of an "item" as a unit of volume (or weight) and of $v_{\max}$ as the vehicle's volume (or weight) capacity. Each destination can then be viewed as a consumption center for packages of unit volume (or weight) – "items" – containing an appropriate product mixture. 当一辆车辆的最大载运体积（或载重）不太依赖于所载货物的种类时，可认为一件货物对应单位体积（载重），$v_{\max}$ 为车辆的最大载运体积（或载重）。每个目的地可视为包含多种类货物合理组合的消费中心。

决策因素

Vehicles are dispatched on service routes from the origin (depot) at times $t_1, t_2, etc.$, on delivery runs to particular subsets of customers (possibly the entire set each time).

- Since vehicles are identical, an operating strategy can be defined relatively easily. We seek the set t_l, as well as the **delivery lot sizes** and the **specific customers** served each "l" ; i.e., the delivery schedules for every customer.
- We also seek **the routes that minimize transportation cost** at each t_l. Our task is simplified because the combined length of all the routes is the main determinant of cost, and simple route length formulas exist

成本构成

- The cost of transportation on **one vehicle route** from one origin to several destinations was approximately a linear function of the **total size of the shipment**, **#. stops** and **the total distance traveled** (recall what we have learn in the lecture on 'Cost').
- If costs on all the vehicle routes are additive, the cost of serving all the destinations for time t_l should be the sum for the costs on each route; i.e., a linear increasing function of the total #. routes (vehicles) used, the total volume shipped, the total number of stops, and the total distance.
- For a given set of delivery schedules to each destination the total volume shipped at each t_l is obviously fixed. Thus, we only need to focus on **#. routes/vehicles, delivery stops, and vehicle-miles when seeking delivery routes for time** t_l.

避免分担运输

- We avoid customer load-splitting among vehicles $\leftarrow$ each destination is visited by the minimum possible number of vehicles able to hold its delivery
    - 1 vehicle if $v < v_{\max}$ items are to be delivered, and $[v/v_{\max}]^+$ otherwise *
- Although in some instances it may be possible to reduce the number of tours and the distance traveled by splitting loads†, the reductions are unlikely to be significant in most cases.
- Among all the possible strategies without load-splitting we prefer the one with the **least distance**, as this strategy should also minimize the number of vehicle routes.
    - A set of routes which minimize total distance should use vehicles to the fullest because fewer line-haul trips to and from the depot then need to be made.

*For customers receiving $v > v_{\max}$ items, one would dispatch $[v/v_{\max}]^-$ full vehicles exclusively to the customer, and would consolidate the remaining items with smaller deliveries to other nearby customers on a single vehicle route.

†see problem 4.1

基本假设

- Since a reasonable set of vehicle routes can be chosen on distance grounds alone, the routes can be designed **without knowing the magnitude of the cost coefficients**.
- Focusing on the difficult case when $v < v_{\max}$, the remainder of this lecture discusses **distance minimizing routing schemes** and presents simple formulas for estimating distance (and therefore transportation costs).
- The results depend on the **number of customers to be served** at time t_l, their **spatial distribution** in the region, and on the **number of stops** that vehicles can make $C = [v_{\max}/v]^-$.
- It is assumed that **the lots carried to each customer are of similar size** (a reasonable assumption for the cases with identical customers discussed in the first few sections of this chapter), so that C is the same for all vehicles; in later lectures, C will be allowed to vary.

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主要参考文献

- Eilon, S., Watson-Gandy, C.D.T. and Christofides, N. (1971) Distribution Management: Mathematical Modelling and Practical Analysis, Hafner, New York, N.Y.
- Daganzo, C.F. (1984a) The length of tours in zones of different shapes, TR-B 18B, 135-146.
- Daganzo, C.F. (1984b) The distance traveled to visit N points with a maximum of C stops per vehicle: An analytic model and an application, TS 18(4), 331-350.
- Newell, G.F. and Daganzo, C.F. (1986). Design of multiple vehicle delivery tours–I: A ring-radial network, TR-B 20B(5), 345-364.
- Newell, G.F. and Daganzo, C.F. (1986a). Design of multiple vehicle delivery tours–II: Other metrics, TR-B 20B(5), 365-376.
- Newell, G.F. (1986). Design of multiple vehicle delivery tours–III: Valuable goods, TR-B 20B(5), 377-390.

Many Vehicle Tours $N/C \gg C$

- In order not to introduce additional notation, we will use N to denote the number of destinations that must be visited. If tours are not being constructed for all the customers in the region, the results can be easily reinterpreted.
- Vehicles should be used to the fullest → there should be at most one vehicle that makes fewer than C stops, and none if N is an integer multiple of C.
- Our strategies are of the "**cluster-first and route-second**" type, where the service region is divided into non-overlapping zones of C customers, to be served by separate vehicles. 先划分服务区域再规划路径
- For a given set of zones, the vehicle routes are easy to construct using some simple rules. To minimize the total distance (and hence the cost), these zones should have specific shapes and orientations, dictated by the relative magnitude of N and C^2 .

Many Vehicle Tours(cont.)

- Two cases need to be considered: (i) when the number of vehicle routes N/C is much greater then the number of stops per route $C, N \gg C^2$, and (ii) when only a few vehicle routes are needed $N \ll C^2$.
- For case (i), delivery districts (or zones) should have a width comparable with the distance between neighboring points and be as long as necessary to contain C points; see Appendix A.
- The formulas are most transparent when expressed in terms of the spatial point density (#.points/unit area) evaluated at a point inside the delivery district, $\boldsymbol{x} : \delta(\boldsymbol{x}) = \mathrm{N}f(\boldsymbol{x})^*$. The factor $\delta(\boldsymbol{x})^{-1/2}$, appearing in the formulas, represents **a distance close to the average separation between neighboring points in the vicinity of $\boldsymbol{x}$.**
- For randomly scattered points, it has been found that (see Appendix A)

$$\text{zone width} \approx (6/\delta)^{1/2}; \quad \text{zone length} \approx C/(6/\delta)^{1/2}$$

*Because $\delta(\boldsymbol{x})$ varies slowly, just like $f(\boldsymbol{x})$, it does not matter which $\boldsymbol{x}$ is used

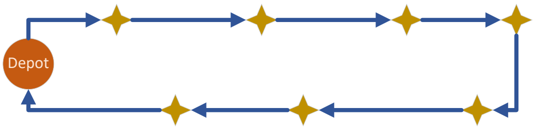
- These dimensions are close to ideal and relatively independent of the metric or underlying network.
- When δ changes over $\boldsymbol{R}$, district dimensions should also change over $\boldsymbol{R}$, although more slowly. As the solution to the EOQ problem, these expressions are robust; departures from the ideal dimensions by 20-30% are largely inconsequential, but larger departures increase distance.
- This robustness makes it easy to carve out $\boldsymbol{R}$ into delivery districts of satisfactory dimensions

服务区域的划分

- Zones should also be oriented “toward the depot”, but the precise meaning of this recipe depends on the underlying metric.
- One should build equi-distance contours from the depot and design zones of the right dimensions that are perpendicular to these contours.
- For the Euclidian metric the contours are concentric circles centered at the depot, so that the zones should fan out from the depot in the radial direction.
- For the L_1 (or “Manhattan”) metric, the contours are squares centered at the depot, at 45° to the metric's preferred directions （等距线应该是以出发点为形心，与量度方向，即 x, y 轴方向成 45° 夹角的一系列正方形）; in this case the zones should be perpendicular to these contours, so that they don't point exactly toward the depot. Ideal orientations can also be defined when the network includes fast/cheap roads.

服务区域的划分

- Because the zones are narrow, it is easy to construct good vehicle routes, once the region has been carved into delivery districts.
- One simply needs to *travel up one side of the zone, visiting the points in order of increasing distance to the depot, and then return along the other side visiting the remaining points in the reverse order*.
- The effectiveness of this routing scheme **improves with the slenderness of the zones –** it is exact if zones are infinitely narrow.

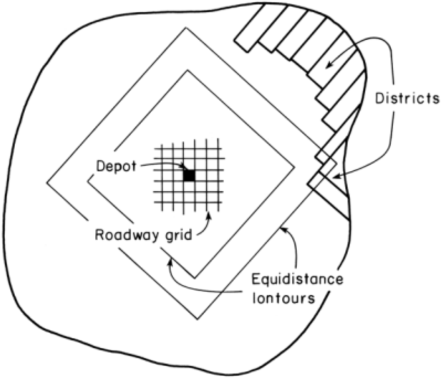


Route in L_1metric in a zone

服务区域划分的例子

Let us show how to partition a region into delivery distances with proper shape and orientation. We may draw delivery zones around the region's edge away from the depot, and then filling in the remaining space with more delivery routes, always proceeding toward the depot.

The figure depicts an intermediate point of this process for an irregular region with an internal depot and a rectangular grid network – note how most districts are perpendicular to the (square) equi-distance (L_1) contours.



As we progress toward the depot, it may become necessary to pack a few zones with the "wrong" shape, but most will have the right dimensions and orientation. Because the distance traveled is not overly sensitive to (small) deviations from the ideal design, the distance formulas about to be developed should be accurate.

单个旅程距离公式

The total distance traveled to visit the C points in a given zone containing point $\boldsymbol{x}_0$ is:

$$\text{tour distance} \approx 2\bar{r} + [k\delta^{-1/2}(\boldsymbol{x}_0)]C,$$

where $\bar{r}$ is the average distance from the C points to the depot (on the shortest path) and k is a dimensionless constant that depends on the metric ($k \approx 0.57$ for the Euclidean metric, and $k \approx 0.82$ for the L_1 metric). See Appendix A for more details.

The first term can be interpreted as the **line-haul distance** needed to reach the center of gravity of the points in the zone from the depot, and the second term as a **local distance** that must be traveled because the points are not next to one another. Note that each stop contributes toward the total a distance comparable with the separation between neighboring points, $k\delta^{-1/2}(\boldsymbol{x}_0)$. This occurs, because the vehicle must be detoured on every leg between successive deliveries. 第一项是从仓库到路径中所有点重心的每个点长途运输距离，第二项为不相邻的点之间的短途运输距离。每个停靠点对总距离的影响是其与相邻节点的间隔距离。

- In actuality, because there are only $C-1$ such legs, the factor "C" in the previous slide should be replaced by "$C-1$". Thus, a better expression is:

$$\text{tour distance} \approx 2\bar{r} + [k\delta^{-1/2}(\boldsymbol{x}_0)](C-1).$$

- The improvement afforded by this expression, particularly obvious for $C=1$, fades in importance as C grows.
- Because the previous equation is more compact, it will be used unless C is small

总距离公式

Let us now see how the total distance over $\boldsymbol{R}$ can be expressed without regard to the detailed position of points, using a continuum approximation.
Distance can be prorated to each one of the points in the zone so that if point i (located at $\boldsymbol{x}_i$) is r_i distance units away from the depot, then:

$$\text{distance prorated to } \boldsymbol{x}_i \approx \frac{2r_i}{C} + [k\delta^{-1/2}(\boldsymbol{x}_0)] \approx \frac{2r_i}{C} + [k\delta^{-1/2}(\boldsymbol{x}_i)]$$

where the second approximate equality follows from the slow varying property of $\delta(\boldsymbol{x})$.
The total distance traveled in the region is the sum of $\boldsymbol{x})i$ across all points:

$$\text{total distance } \approx \frac{2}{C}\sum_i r_i + k\sum_i \delta^{-1/2}(\boldsymbol{x}_i)$$

For large N, the sum can be replaced by integrals independent of the specific location of all the points:

$$\sum_i \delta^{-1/2}(\boldsymbol{x}_i) \approx \int_{\boldsymbol{R}} [\delta^{-1/2}(\boldsymbol{x}_i)] \mathrm{d}\boldsymbol{x}, \quad \text{and} \quad \sum_i r_i \approx \int_{\boldsymbol{R}} r(\boldsymbol{x})\delta(\boldsymbol{x})\mathrm{d}\boldsymbol{x}$$

Thus

$$\text{total distance } \approx \int_{\boldsymbol{R}} [\frac{2}{C} r(\boldsymbol{x}) + k\delta^{-1/2}(\boldsymbol{x})]\delta(\boldsymbol{x})\mathrm{d}\boldsymbol{x}.$$

Note that this expression is well suited for continuum approximations because the cost in any given (small) area only depends on the local conditions

Alternative Expression

An alternative expression for the total distance is obtained after replacing $\delta(\boldsymbol{x})\mathrm{d}\boldsymbol{x}$ by $Nf(\boldsymbol{x})\mathrm{dx}$, it then becomes clear that these expressions can be interpreted as the product of N and the expectation of $r(\boldsymbol{x})$ or $\delta^{-1/2}(\boldsymbol{x})$, when the probability density of position is $f(\boldsymbol{x})$. Thus, letting $E(r)$ and $E(\delta^{-1/2})$ denote these expectations, the total distance can be expressed as:

$$\text{total distance } \approx N[\frac{2E(r)}{C} + kE(\delta^{-1/2})].$$

For a uniform density, $E(\delta^{-1/2}) = \delta^{-1/2} = \sqrt{|\boldsymbol{R}|/N}$ and we can write:

$$\text{total distance } \approx N[\frac{2E(r)}{C} + k\sqrt{|\boldsymbol{R}|/N}.$$

where $|\boldsymbol{R}|$ denotes the surface area of $\boldsymbol{R}$.

Interpretation

- Independent of the specific locations, *these equations are particularly useful if cost must be estimated before the point locations are known*. In that instance, it may be reasonable to view the actual locations $(\boldsymbol{x}_1, \ldots, \boldsymbol{x}_N)$ as outcomes of i.i.d random variables with density $f(\boldsymbol{x})$, and interpret the following equation as the average total distance over all possible locations $(\boldsymbol{x}_1, \ldots, \boldsymbol{x}_N)$

$$\text{total distance } \approx N[\frac{2E(r)}{C} + kE(\delta^{-1/2})].$$

- In any specific instance there will be some discrepancy between the equation and the actual distance — for large N most of the difference typically will arise from fluctuations in $\sum_i r_i$, which are of order $O(N^{1/2})$ and comparable to the second term. If more accuracy is desired, one should wait for the point locations to become known. Comparisons made in Hall et. al. (1994) indicate that the approximation formulas just presented are fairly accurate even if the number of stops is not the same for all tours.

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Few Vehicle Tours $N/C \ll C$

- If $C^2 \gg N$, the optimal strategy must be different from the one we just explained because zones of ideal length (approximately $\frac{C}{(6\delta)^{1/2}}$ would be too long to fit in the service region.
- It is not too difficult to design a partition of the region that will yield a distance close to a lower bound for the optimum; i.e., a near-optimal partition.
- **The lower bound is the distance for the shortest single tour visiting all the points, beginning and ending at the depot** — the "traveling salesman problem" (TSP) tour. Before describing the partitioning strategy, we must learn some basic properties of TSP tours with many points.

Few Vehicle Tours $N/C \ll C$ (cont.)

If a region with a nearly constant density of points is partitioned into a few subregions with many points each, then the length of the shortest tour in the region is close to the sum of the optimal subregional tours.

- a grand tour can be constructed by connecting the optimal tours of the subregions with a few new legs, while at the same time deleting a like number of existing legs → 将各个子区域的最优旅程通过若干小路程相连，同时删去若干小路程，即可获得一个连接所有点的旅程
- subregional tours can be constructed from a grand optimal single tour, by connecting the broken sections of the grand tour within each subregion with legs along its boundary.→ 将穿过所有点的单个最优旅程中穿过各子区域的部分的边界通过小路程相连，即可获得各个子区域的旅程

In both cases, the original (optimal) and modified (suboptimal) tours differ in total length by no more than the **combined perimeter of all the subregions**, which is a relatively small quantity when the number of points is large. Thus, the optimal grand tour should be just about as long as all the optimal subregional tours combined.

*Karp, 1977, and Eilon et al. 1971. See Appendix A for a simple proof.

Few Vehicle Tours $N/C \ll C$ (cont.)

- This property suggests that if the density of points is constant, then the TSP tour for a subregion 1/4th the region's size (with 1/4th the points) should be about four times shorter; that is, the average distance per point should be roughly constant. Since the only distance parameter of the problem is $\delta^{-1/2}$, the distance per point for large N must be of the form: $k'\delta^{-1/2}$, where k' is a dimensionless constant, independent of region shape but dependent on the metric; k' is believed to be about 0.75 for the Euclidean metric with randomly distributed points.
- The expression also holds, with a different k', for regular arrangements of points. Note that the total tour distance can be expressed as: $k\sqrt{N|\boldsymbol{R}|}$.

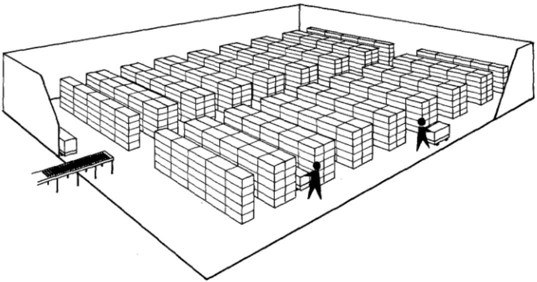
- In light of the TSP tour partitioning property, it should not matter much how the region is partitioned for the vehicle routing problem (VRP), provided that travel external to the districts is avoided by ensuring that every zone touches the depot.
- In that case each VRP tour will be similar to the TSP in the district (the TSP may not have to visit the depot), and the combined VRP length should be close to the overall TSP length; i.e., the lower bound. This means that traditional "sweep"-type algorithms for the VRP, which result in wedge shaped districts as we desire, should work well for the case with $N \ll C^2$.

- Alternatively one can build a TSP for the whole region, $\boldsymbol{R}$, and partition it into segments of C points each that would be connected to the depot.
- The length of these segments is negligible compared to the total (if $N \ll C^2$), so that the length of all the tours should be close to the length of a TSP.
- In either case, the length of all tours is close to the TSP lower bound. If the density is constant, we can write:

$$\text{total distance} \approx k' N \delta^{-1/2} = k' \sqrt{N|\boldsymbol{R}|}.$$

System with blockages

- As an aside we note that these equation, which rest on partitioning properties of TSP and VRP tours, may need to be modified for systems in which the distance metric cannot be used to define a "norm".
- An example of such a metric is a rectangular warehouse with a system of transversal aisles (横断面通道) that block travel in the longitudinal direction（纵向）, except along the sides of the rectangle.



System with blockages (cont.)

- For this type of system the length of a tour in which all the aisles with one or more service points are traversed in succession is (Kunder and Gudeus, 1975):

$$\text{total distance} \approx 2y_1 + ay_2$$

  where y_1 and y_2 are the longitudinal and transversal dimensions of the rectangle, and a is the number of aisles containing a point. y_1, y_2 分别是纵向和横向的长度，a 是包含需要访问的点的通道个数
- If N is so large that each aisle contains many points it should be clear that: (i) the traversal strategy becomes optimal and (ii) the coefficient a of the above expression can be replaced by the number of aisles.

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*This shows that the above expression is not of the form since both its terms are independent of N for $N \to \infty$.

- We now return to the (usual) cases where the TSP/VRP formula can be applied, and note that for slow-varying nonuniform densities this distance expression can be approximated by the sum of expected TSP lengths over subregions with many points and (nearly) constant density. In integral form this is:

$$\text{total distance} \approx k' N E(\delta^{-1/2}).$$

where $E(\delta^{-1/2})$ is defined previously. The uniform density is proofed to maximize the total distance; thus $k'\sqrt{N|\boldsymbol{R}|}$ is an upper bound to $k' N E(\delta^{-1/2})$.

- Notice that, unlike the many tour case, these equations are independent of C; i.e., if vehicles make so many stops that zones of ideal length cannot be packed in the service region, then travel distance is not decreased appreciably by increasing C.

- The vehicle routes within the wedge shaped zones are more difficult to develop in this case than in the previous one, which should not be surprising since the TSP problem is NP-hard.
- Nonetheless, simple algorithms such as the ones described in Daganzo (1984a) and Platzman and Bartholdi (1989) can yield tours within 20% of optimality. Simple fine-tuning corrections (see Newell and Daganzo, 1986) can then reduce its length by another 10 or 15%. Other fine-tuning approaches can yield tours even closer to optimality (see Robusté et al. 1990).
- It is not our purpose to describe here existing tour construction methods, since this is of marginal value for the theories that will follow. Suffice it to say that, in practice, it is possible to obtain tours within a few percent of optimality with an effort that only grows proportionately with the number of points to be visited.

- Now return to the one-to-many problem with identical customers.
- Recall that we are seeking the set of delivery schedules for each customer and that, given the schedules, the transportation cost at each t_l can be easily estimated with the results that have just been presented.
- The chosen schedule should strike the best balance between transportation and holding costs

Any questions?

Readings

- Daganzo. Logistics System Analysis. Ch.4. Page 93-104